Questions for state doctoral exam – field of study: Chemical and Process Engineering
Subtopic: Dynamics

1. Definition of a dynamical system with continuous and discrete time. Conservative vs dissipative systems, Liouville theorem. State (or phase) space, basic types of asymptotic dynamics, examples: Lorenz model, Henon model, logistic equation with time delay.

2. Dynamics of systems with continuous time. Vector field, trajectory/orbit, flow. Asymptotic dynamics, invariant sets, attractors and their characterization using dimension.

3. Steady/stationary state and its stability. Invariant manifolds of the steady states, analysis using linearization in the neighborhood of the steady state. Jacobi matrix, eigenvalues and eigenvectors, stable, unstable and central linear subspace, time evolution of a perturbation, types of phase portraits in the neighborhood of the steady state.

4. Periodic trajectory, its stability. Linearization of a dynamical system in the neighborhood of the periodic trajectory, monodromy matrix. Floquet multipliers. Saddle loop. Poincaré mapping. Periodic orbits in systems with discrete time.

5. Deterministic chaos. Sensitive dependence on initial conditions, chaotic attractor and measures of its internal instability and complexity: fractal dimension, Lyapunov exponents, Lyapunov dimension. Characterization of attractors using the spectrum of Lyapunov exponents.

6. Structural stability, dependence of dynamics on parameters. Local bifurcations of steady states/points and periodic trajectories.

7. Description of the bifurcation sequences leading to chaotic dynamics. An example with discrete time: logistic map, period-doubling sequences, Feigenbaum constant, structure of periodic windows, reverse sequences of chaotic attractors, intermittent chaos, metastable dynamics.

8. Dynamics of oscillatory systems with periodic pulsed perturbations. Description using Poincare map, discretization in time, and phase transition curves. Rotation number, resonant periodic trajectories, regions of resonance in a space of two parameters – the amplitude and period of the pulses. Occurrence of periodic, quasiperiodic and chaotic dynamics in the parameter plane, characterization using rotation number (“devil’s staircase”) and Lyapunov exponent.