Sedimentation

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I Fundamental relations and definitions

Settling, or sedimentation serves to separate solid particles from liquid in the gravitational field. The density of particles must therefore differ from the density of fluid. This difference in densities determines in which direction the sedimentation occurs and determines to a great extent (among other factors) the velocity of particles. The sedimentation velocity is an essential component of designing sedimentation equipment (e.g. gravitational sedimentation columns, either continuous flow or periodic).

The sedimentation velocity of an isolated particle will differ from that of the same particle when it settles in the presence of other particles, due to the mutual interaction between them. This case is designated as “disrupted” sedimentation. When designing sedimentation columns, however, we often neglect the mutual interactions of particles, especially at low concentrations and consider the sedimentation as undisrupted.

We will consider ***the motion of isolated particle in a stationary*** fluid medium. For the case of positive orientation in the direction of gravitational force, we can describe the motion according to Newton’s second law for one-dimensional motion in scalar terms (all forces act in the vertical direction), in the form;

$$V_p \rho_p g - V_p \rho g - \mathrm{sign}(\upsilon)\, \zeta_u\, S_p \rho g \frac{\upsilon^2}{2} = V_p \rho_p \frac{d\upsilon}{d\tau} \tag{7-1}$$

Where V_p is the volume of particle, S_p - the cross section of particle perpendicular to the difrection of flow, ρ_p and ρ - densities of particle and fluid medium, g gravitational acceleration, υ - instantaneous velocity of particle with respect to the surrounding fluid and ζ_u - resistance coefficient of medium against motion of particle.

The term on the right hand side of Eq. (7-1) expresses the time dependency of particle momentum. On the left hand side are sequentially expressed forces acting on particle: gravitational $F_G = m_p\, g$, buoyancy or Archimedes force $F_A = V_p\, \rho\, g$ and the resistance force of the medium $F_R = \zeta_u\, S_p\, \rho\, \upsilon^2/2$, which always acts against the velocity of particle; this fact is expressed by the quantity sign(υ), which is equal to +1 in the case of positive particle motion (chosen as the direction of gravitaional force), or is equal to -1 in case of opposite direction of particle velocity.

In typical sedimentation columns, only a short time elapses (fractions of a second) from the start of the process to an equilibrium steady state of the forces acting on the particle, when the resultant of all the forces is equal to zero. Acceleration is therefore also zero and the particle moves uniformly onwards. If we assume that in Eq. (7-1), the density of particle is greater than the density of fluid ($\rho_p > \rho$), the particle will move in the direction of gravitaional acceleration at constant velocity, which we will call ***sedimentation velocity*** $\boldsymbol{\upsilon_u}$.

From Eq. (7-1) we get

$$V_p\, g\, (\rho_p - \rho) = \zeta_u\, S_p\, \rho \frac{\upsilon_u^2}{2} \tag{7-2}$$

We further express the volume V_p and area S_p for the case of ***spherical particles*** by the equations

$$V_p = \frac{\pi d_p^3}{6}, \qquad S_p = \frac{\pi d_p^2}{4} \tag{7-3}$$

where d_p is the diameter of particles determined by direct measurement or by sieve analysis etc. After combining Eqs. (7-2) and (7-3) we obtain

$$g\ d_p^3 (\rho_p - \rho) = \frac{3}{4} \zeta_u\ \rho\ d_p^2\ \upsilon_u^2 \tag{7-4}$$

to determine the sedimentation velocity υ_u from this equation, however, we need to know the resistance coefficient ζ_u by experiment or calculation. It was found that the dimensionless quantity ζ_u depends on the sedimentation velocity and the particle diameter, density and viscosity of medium as well as on the geometrical arrangement of apparatus. From Eq. (7-4) a definition of the resistance coefficient can be obtained:

$$\zeta_u = \frac{4}{3} \frac{g d_p}{\upsilon_u^2} \frac{\rho_p - \rho}{\rho} \tag{7-4a}$$

In sedimentation, just as in other hydrodynamic process, a ***Reynolds number*** is introduced Re_u by the relation

$$\mathrm{Re}_u = \frac{\upsilon_u\ d_p\ \rho}{\eta} = \frac{\upsilon_u\ d_p}{\nu} \tag{7-5}$$

where η designates the dynamic and ν the kinematic viscosity of medium. For the kinematic viscosity applies $\nu = \eta / \rho$.

the dependency of ζ_u on Re_u was found experimentally and can often be approximated by a power function

$$\zeta_u = \mathrm{A}\ Re_u^{a} \tag{7-6}$$

where the numerical constants A and a attain different values in relation to the character of circumfluence of particle by fluid and were also determined experimentally. In literature, this experimentally determined dependency $\zeta_u(Re_u)$ is often illustrated graphically.

The calculation of sedimentation velocity υ_u from Eqs. (7-4, 7-5, and 7-6) needs to be done by iteration, because the sedimentation velocity υ_u is contained in the definition of the resistance coefficient ζ_u as well as in Reynolds' number Re_u.

From theory, it can be seen that the description of sedimentation of particles by the dimensional Eq. (7-4) can be generalized by transforming to a dimensionless form and having a dependency of only *two* criteria; e.g. in Eq. (7-6) these criteria are the resistance coefficient ζ_u and the Reynolds number Re_u. However, for practical calculations in sedimentation, the criteria are defined in such a way that the most often calculated quantities, i.e. the sedimentation velocity υ_u and diameter of particles d_p do not appear simultaneously (iteration is thus eliminated).

One of these criteria is the ***Archimedes number*** Ar, which is defined as:

$$Ar = \frac{g\,d_p^3}{\nu^2}\frac{(\rho_p - \rho)}{\rho} = \frac{g\,d_p^3 \rho (\rho_p - \rho)}{\eta^2} \qquad (7\text{-}7)$$

Further, the ***Lyashchenk number*** $Ly = Re^3 / Ar$ is introduced, which, from the calculated quantities, contains only the sedimentation velocity. Thus it applies;

$$Ly = \frac{\upsilon_u^3 \rho^2}{g\,\eta (\rho_p - \rho)} \qquad (7\text{-}8)$$

Instead of Eq. (7-6), it is therefore possible to express the sedimentation equation as a dependency of Ly and Ar in a similar power equation:

$$Ly = \mathrm{B}\, Ar^{\mathrm{b}} \qquad (7\text{-}9)$$

which is now suitable for the direct calculation of either the sedimentation velocity υ_u (contained only in the Lyashchenk number) or the particle diameter (contained only in the Archimedes number).

Constants B, b in Eq. (7-9) are again dependent on the character of circumfluence of particle and their values, obtained by regression analysis of experimental data for three different regions of circumfluence are presented in table 7-1.

Table 7-1 values of constants in Eqs. (7-6) a (7-9)

Character of circumfluence for spherical particles		**laminar** *(Stokes region)*	**transitional** *(Allen region)*	**turbulent** *(Newtonian region)*
Limiting values of criteria	Re_u	$\le 0{,}2$	$0{,}2 - 5{,}0 \cdot 10^2$	$5{,}0 \cdot 10^2 - 1{,}5 \cdot 10^5$
	Ar	$\le 3{,}6$	$3{,}6 - 8{,}4 \cdot 10^4$	$8{,}4 \cdot 10^4 - 7{,}4 \cdot 10^9$
Values of constants in Eqs. (7-6) a (7-9)	A	24	18,5	0,44
	a	-1	-0,6	0
	B	$1{,}71 \cdot 10^{-4}$	$3{,}56 \cdot 10^{-3}$	5,27
	b	2	1,14	0,5

Comparison of measured values and literature values. For calculating the quantities υ_u or d_p, it is also possible, besides using Eq. (7-9), to use the graphical dependence $Ly^{1/3}(Ar^{1/3})$ on Fig. 7-1. For spherical particles, the curve designated by parameter $\psi_V = 1$ applies.

The description of ***sedimentation of non-spherical particles*** is generally more complex than for spherical particles. Other factors come into effect: shape of the particle, orientation and deviation from vertical motion. Certain generalizations are possible for so-called ***isometric particles***, whereby, the length coordinates in mutually perpendicular directions are roughly similar (e.g. a cube). For these, curves designated by the parameter $\psi_V < 1$ are drawn in Fig. 7-1.

The sedimentation velocity is then determined analogically to spherical particles, with the only difference that we substitute the diameter with their ***equivalent diameter*** $\boldsymbol{d_{ek}}$, which is expressed by the relation

$$d_{\text{ek}} = \left(\frac{6 V_{\text{pn}}}{\pi} \right)^{1/3} \tag{7-10}$$

whereby V_{pn} is the volume of the non-spherical particle. The definition of d_{ek} is chosen such that for a sphere $d_{\text{ek}} = d_{\text{p}}$. The quantity ψ_V is called the ***sphericity of non-spherical particles*** and is defined by the relation

$$\psi_V = \frac{\pi d_{\text{ek}}^2}{A_{\text{pn}}} \tag{7-11}$$

where A_{pn} is the surface area of the non-spherical particles.

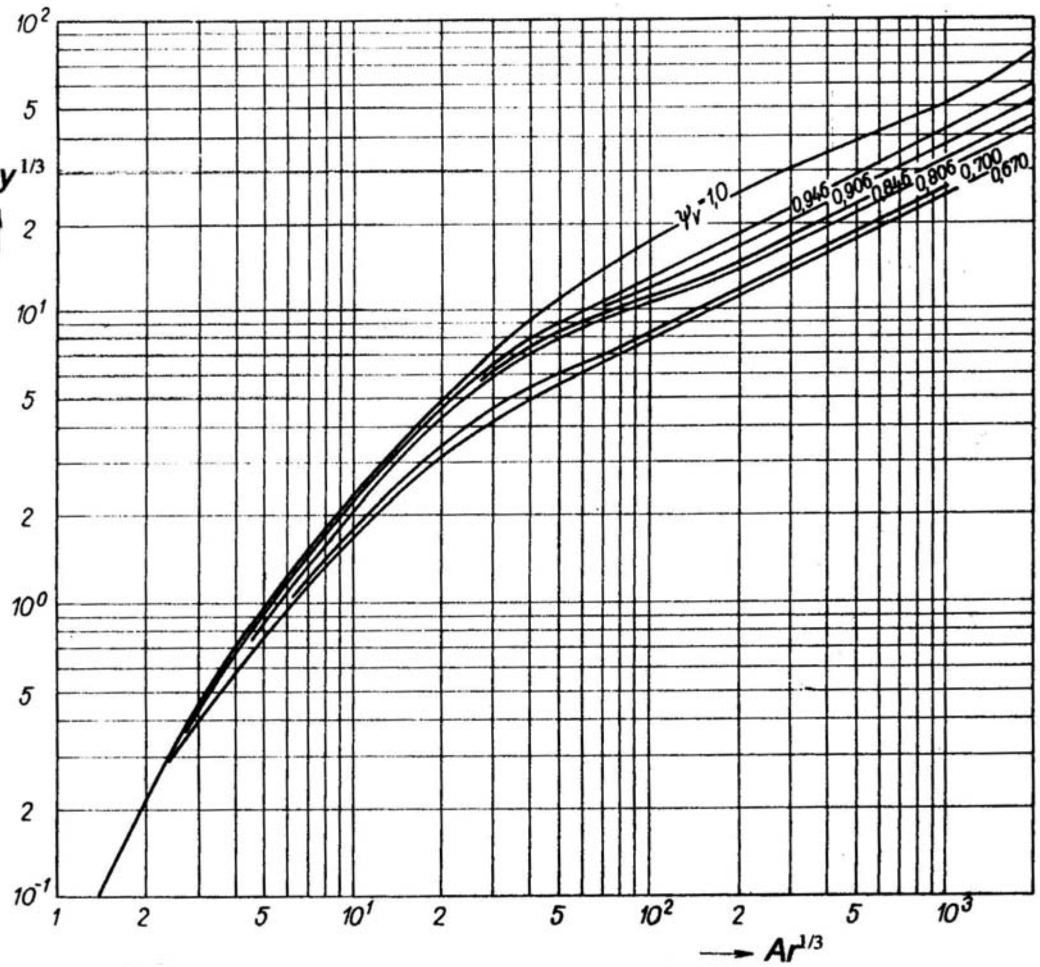


Fig. 7-1 Dependency of $Ly^{1/3}$ ($Ar^{1/3}$, ψ_V) for the sedimentation of spherical and non-spherical particles in the transitional and turbulent regions. The parameters of the curves is the sphericity ψ_V

II Aim of the Work

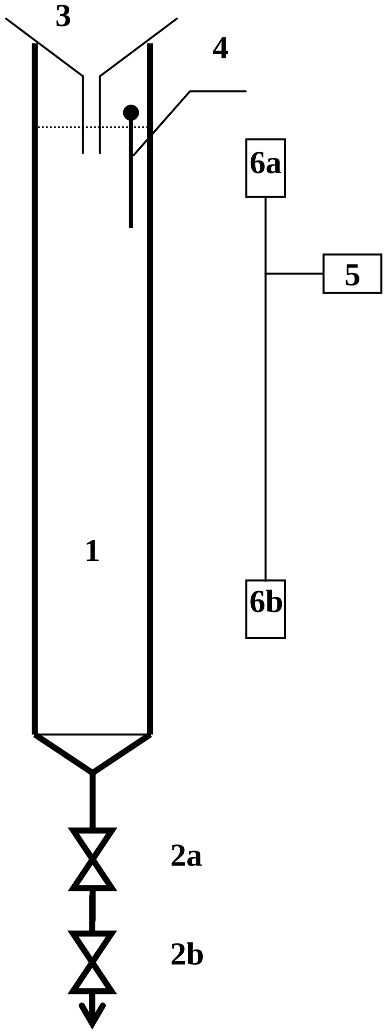
1. measuring the sedimentation velocity of spherical and non-spherical particles of different sizes under constant physical conditions of liquid, calculating the average and the standard deviation of sedimentation velocity
2. compare the measured average values of sedimentation velocity of spherical particles with the values calculated from the graph on Fig. 7-1 and from Eq. (7-9)
3. compare the measured average values of sedimentation velocity of non-spherical particles with values calculated for using the sphericity

4. measuring the viscosity and density of liquid medium

III Description of apparatus

The sedimentation apparatus is schematically drawn in Fig. 7-2. The glassy tube (column) **1** of diameter 100mm is at the bottom equipped with two glass taps **2a, b**, which serve to empty the column and catch the particles. At the bottom and top end of the column, there are two scale lines, which are at a distance 2.4m from each other. The particles are fed into the column **1** by funnel **3**, at the top end, and a thermometer **4** measures the temperature of the liquid.

Further, the apparatus is equipped with a stopwatch **5**, for measuring the sedimentation time of particles. The stopwatch is controlled by two switches, **6a, b**, located at the top and bottom ends of the column.



1 – glass tube (column)
2 a,b - glass taps
3 glass funnel
4 - thermometer
5 digital stop watch
6 a,b - stop watch switch

Fig. 7-2 Scheme of the apparatus for measuring the sedimentation velocity of particles

Some integral components of the apparatus are also; the Hoppler viscosity meter (see Fig 7-3) with a thermostat and accessories, a density meter and assorted particles whose sedimentation velocity will be measured.

IV Experimental procedure

IV.1 Preparation of the particles

For measuring the sedimentation velocity, particles pre-sorted according to their shape or sizes are used. For spherical particles, the diameter and density are given; for non-spherical particles, the volume V_{pn}, surface area A_{pn}, largest linear dimension $l_{p,max}$ and the density are given. All these quantities are included in the table, which is hanging just by the measuring apparatus.

Before the experiment, we should place the clean particles into a beaker with a small amount of sampled solution and during experiment they are freely dropped by tweezers or individually discharged with a small amount of solution into the glass funnel, whose stem is submerged in the column liquid.

IV.2 Actual measurements

Before the start and at the end of the measurements with each set of particles, read off the temperature of the solution in the tube for thermometer **4** and write it down in the protocol.

During the experiment, we determine the time, which elapses when the particle passes between the two horizontal scales indicated at the top and bottom end of the column. Before releasing the particle, we should open the tap **2a**; the sedimented particles gather in the space above the closed tap **2b**. Measurement is performed by two observers. The top one starts the stopwatch at the moment when the particle passes the upper scale line by pressing the button START an the switch **6a**. At the same time, for the information of the bottom observer, the signal on switch **6b** lights up. The bottom observer stops the stopwatch by pressing STOP when the particle passes the bottom scale line. The particles are released into the column individually and besides measuring the time, the observers should also observe the trajectory of the particle. In some cases, the particles can move along the column wall or they can even make contact with it. The times for these cases should be clearly marked and excluded from the calculations. It is necessary to obtain at least 15 regular trajectories for a given set of particles. After writing down the sedimentation time, the stopwatch should be zeroed by the button “0” on the sensor **6a**. If need be, used particles can be withdrawn from the column; the glass tap **2a** is first closed and by subsequently opening the tap **2b**, we can discharge the particles via a sieve to a prepared collecting vessel. After measurement of half of the given set of particles has been carried out, it is prudent for the observers to exchange positions; this reduces the subjective measurement error caused by different reaction times of the observers.

IV.3 Finishing off the experiment

After completing the measurements, the sedimented particles are then discharged from the column, washed with water, dried and put into containers. The remaining uncontaminated solution, collected as a sample in the collecting vessel during the discharging of particles, should be poured back into the column from above.

A wet cloth should be used to clean all the soiled places. The bottom end of the column, floor and wooden tile should be washed with water and dried by a cloth.

IV.4 Determining the physical properties of liquid

The measurement of density and viscosity is also included in the experimental work.

IV.4.1 Determining the density

In this work, the density is measured on one hand by a submerged glass density meter in a measuring cylinder which is tempered by standing it in the thermostat and on the other hand by a digital density meter.

During the work with the submerged glass density meter, it is cumbersome to precisely set the temperature of investigated liquid to the exact temperature in the sedimentation column; hence we calculate the density from a proportionality constant E.

Within the range of temperatures existing in the column during experiment, we can assume that the density is a linear function of temperature. Then the following equation applies

$$\rho(t) = \rho(t_1) + E\,(t - t_1) \qquad (7\text{-}12)$$

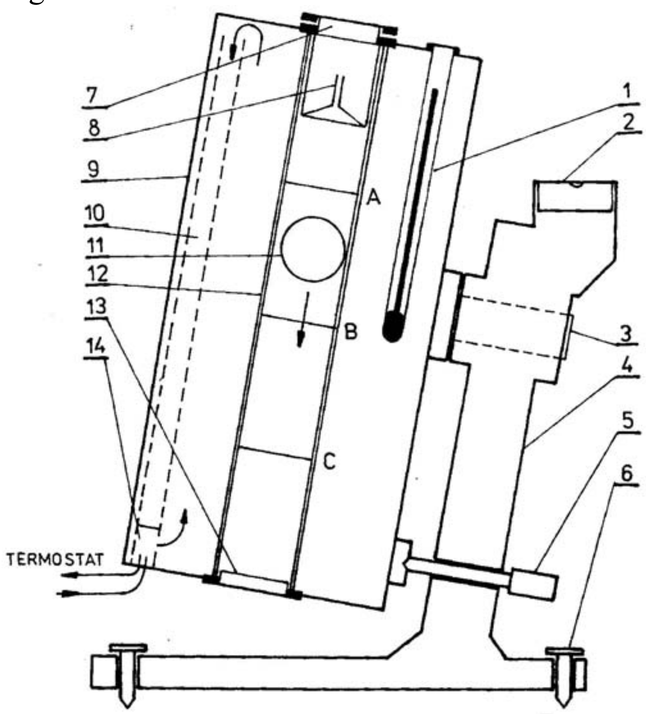
where $\rho(t)$ is the density at temperature t, $\rho(t_1)$ is the density at the set temperature t_1 and E is a coefficient of proportionality. Physically, the coefficient E expresses the change of density per 1°C temperature change, as can be seen from its units $[E]$ = kg m^{-3} K^{-1}.

To determine the density at an arbitrary temperature, it is enough therefore to know the density at one temperature and the coefficient E. According to Eq. (7-12) we determine the value of E by measuring the density of liquid at two different temperatures t_1 and t_2, which we choose when tempering the sample temperature and we set the thermostat in such a way that the difference between these set temperatures is about 10°C and such that the mean temperature of solution t in the experimental column during experiment lies within the interval t_1 and t_2. During experiment, we neglect the change in the volume of the density meter with temperature. If necessary, during tempering, the solution in the measuring cylinder can be mixed by careful movement of the digital density meter.

The digital density meter automatically tempers the sample temperature according to the set temperature; therefore we could measure the density at the actual solution temperature. The instructions for operating the digital density meter are available in the laboratory.

IV.4.2 Determining the dynamic viscosity

The viscosity of solution η is measured in the laboratory by a Hoppler viscosity meter with a tempering circuit connected to the thermostat. The scheme of the apparatus is shown in Fig. 7-3.



1 thermometer
2 leveller
3 swivel
4 stand
5 holding bolt
6 standing nuts
7 upper plug
8 de-aerating cap
9 sheath
10 output of thermostat.liquid
11 sphere
12 tube with scale levels
13 lower plug
14 input of thermostat.liquid

Fig. 7-3. Scheme of the Höppler viscosity meter

The size of the sphere **11** and the diameter of the tube **12** is generally selected in such a way that the liquid flow in the space between the sphere and the tube walls is laminar, the sphere flows uniformly downwards under the influence of gravity

Before measuring, using nuts **6** we should adjust the viscosity meter to a horizontal position according to the leveller **2**. We then connect the viscosity meter to the thermostat and set the required temperature. Clean the tube and the sphere, rinse with water and finally with measured liquid. We fix the lower plug **13**, pour the sample liquid to the scale level **C** and drop the appropriate sphere. After this we fill the tube up to about 1cm below the upper scale limit. We then insert the de-aerating cap **8** (the tube is closed without bubbles since the extra liquid is forced into the cap). Finally we close the tube using plug **7**. We should turn the lids carefully, so as not to break the tube. After this, the device is prepared for measurement. By turning the viscosity meter about the swivel **3**, we place the sphere above scale level **A**. By reverse turning and securing with bolt **5**, we commence the measurement. We measure the time required for the sphere to pass through the scale levels. We usually use the scales **A** and **C**. the precision of measurements is given as at 0.5% to 1%. The drop time should not be shorter than 30s (for a trajectory AC = 100mm).

We measure the viscosity at two selected temperatures t_1 and t_2. The dependency of viscosity on temperature can be generally expressed by the exponential function

$$\eta = C \exp(D/T) \qquad (7\text{-}13)$$

⇨**<u>Calculation procedure:</u>** For the average values of the sphere drop time τ_1 and τ_2 at two selected temperatures t_1 and t_2 we calculate the dynamic viscosity of solution η_1 and η_2 from the equation:

$$\eta = K\,(\rho_k - \rho\,)\,\tau \qquad (7\text{-}14)$$

where K is the sphere constant, ρ_k is the density of the sphere (both are given in the sign hanging by the apparatus). These viscosities, together with the appropriate temperatures in K are then fed into Eq. (7-13), hence we obtain two equations for two unknown constants C and D, which we solve. Substituting these constants and the average temperature of the solution into Eq. (7-13) will then give us the sought value of viscosity of the solution.

V Safety precautions

1. While working with the viscosity meter, it is of paramount importance to proceed according to the instructions in section IV.4.2, especially not forcibly tightening the plug lids of the tube. The sphere should not be dropped to the floor or anywhere - it is polished to a precision of 1μm
2. In all samplings and working with the solution outside the apparatus, please practice cleanliness. The uncontaminated solution is returned back to the column. The price of 1L of glycerol is above 210Kč.
3. If any solution is spilt, then the area should be cleaned with a cloth. There is a danger of slipping and falling.

VI Processing the measured data

VI.1 Evaluating the experimental results

During experiment with a set of particles, a set of regular measurements of the sedimentation time of particles τ through a distance of length L designated between the two scale lines was obtained. The sedimentation time in which the particle moves along the column wall or touches it were specially marked and excluded from all other calculations.

Note: to be precise, it would be necessary to consider the influence of column wall in every measurement, because the walls reduce the sedimentation time. In our experiments, we can afford to neglect it because the ratio of particle diameter to column diameter if very small

For a given set of particles, we calculate and write down in the protocol the sedimentation time of individual measurements according to the equation for uniform straight motion

$$\upsilon_{ui} = L / \tau_i \qquad (7\text{-}15)$$

For a set of these velocities, υ_{ui} according to chapter 2 on Data Processing, we calculate

a) the sample average $\overline{\upsilon}_u$,
b) the standard deviation $s_{\upsilon u}$,
c) the confidence interval for $\overline{\upsilon}_u$.

VI.2 Calculating the sedimentation velocity from criterial relations

Into all the used criteria, we should enter in the values of density ρ and viscosity η at the mean temperature of solution, which is determined as the arithmetic mean between the temperatures at the start and end of experiment.

For the calculation of ρ and η at this mean temperature, we use the values of E and constants C and D, determined from relations and instructions in section IV.4.1 and 2.

A. For **spherical particles**, calculate the sedimentation velocity on one hand using Eq. (7-9) and on the other hand using the graph in Fig. 7-1.
   In the first method of calculation, we choose from table 7-1 the appropriate constants B, b according to the value of Ar calculated from Eq. (7-7); from Equation (7-1) we calculate the Lyashchenk number Ly and express υ_u from Eq. (7-8).
   In the second method of calculation, for the appropriate value of $Ar^{1/3}$ and from the curve with the parameter $\psi_V = 1$ in Fig. 7-1, we directly determine the value of $Ly^{1/3}$ and calculate the value of υ_u again from Eq. (7-8).
   Compare both values of sedimentation velocity from the calculations with the value of average experimental sedimentation velocity $\overline{\upsilon}_u$.
B. For **non-spherical particles**, we perform the calculation of sedimentation velocity using the sphericity ψ_V. First we calculate the d_{ek} from Eq. (7-10) and for this diameter we then calculate Ar from Eq. (7-7).
   The sphericity ψ_V is calculated from Eq. (7-11) and in the graph on Fig. 7-1 for the curve with the calculated value of ψ_V (the curve with the appropriate sphericity with probably have to be obtained by interpolation) we read off the value of $Ly^{1/3}$.
   The sedimentation velocity is then calculated from Eq. (7-8).

VII List of symbols

A, a	Constants in Eq. (7-6)	
A_{pn}	Surface area of non-spherical paricles	m^2
Ar	Archimedes number, Eq. (7-7)	
B, b	Constants in Eq. (7-9)	
C, D	Constants in Eq. (7-13)	
d_{ek}	equivalent diameter	m

d_p	diameter of spherical particles	m
E	coefficient defined in Eq. (7-12)	kg m^{-3} K^{-1}
K	constant of the viscosity meter sphere	
L	distance between the scale lines in the column	m
$l_{p,max}$	largest linear dimension of non-spherical particles	m
Ly	Lyashchenk number, Eq. (7-8)	
Re_u	Reynolds number for sedimentation, Eq. (7-5)	
S_p	cross section of particle perpendicular to the plane of particle motion	m^2
V_p, V_{pn}	volume of spherical, non-spherical particles	m^3
υ	instantaneous velocity of particle with respect to the medium	m s^{-1}
υ_u	sedimentation velocity of particle	m s^{-1}
$\overline{\upsilon}_u$	sample average sedimetation velocity of particles	m s^{-1}
ζ_u	coefficient of resistance against the motion of particle	
ρ_p	density of particle	kg m^{-3}
ψ_V	sphericity of non-spherical particles, Eq. (7-11)	

VIII Control questions

1. What is the difference in the calculation of sedimentation velocity between calculation using Eq. (7-9) and calculation using Fig. 7-1?
2. What is the sphericity?
3. Explain the principle of measuring the viscosity using the Hoppler viscosity meter. Which other methods of determining viscosity do you know?
4. Explain the principle of measuring the liquid density by the density meter. Which other methods do you know?