Heat transfer in a fin heat exchanger

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Basic relations and definitions

In cases when heat transfers from medium with high heat transfer coefficient into a medium with low heat transfer coefficient, the size of the heat exchanger is given by the lower value of the heat transfer coefficients. If the heat exchanger was made of smooth heat exchanging surfaces, the exchanger would be too large. One solves this problem by increasing the heat exchange area only on the side of the medium with higher heat transfer resistance (low heat transfer coefficient). The easiest solution is mounting fins to the appropriate side of the heat exchanger. Typical examples of this arrangement are warm-air furnaces, i. e. heat exchangers in which gas heats up usually with condensation heat of steam. When designing a warm-air furnace we start with the equation:

$$\dot{Q} = k_{\mathrm{Z}} A \Delta t_{\mathrm{ls}} \,, \tag{1}$$

where $\dot{Q}$ is the heat flow, k_z is the overall heat transfer coefficient for exchange surface with fins, A is the heat exchange area and Δt_{ls} is the logarithmic mean temperature difference defined as the difference of temperatures of warmer medium t_{A} and cooler medium t_{B} at the ends of the heat exchanger (see equ. 9). The relations for calculating the value of k_z vary based on the geometrical arrangement and the conditions of fluid B flow.

The apparatus installed in the laboratory consists of a tube bundle with radial ring fins of constant thickness. The tubes are oriented perpendicularly to the gas flow (medium B). The following expression can be used for such a case:

$$\frac{1}{k_Z} = A_{\mathrm{C}}\left[\frac{1}{\alpha_{\mathrm{Z}}(A_{\mathrm{C}} - (1-\Omega)A_{\mathrm{Z}})} + \frac{1}{A_{\mathrm{i}}}\left(\frac{1}{\alpha_{\mathrm{i}}} + \frac{\delta_W}{\lambda_W}\right)\right] \tag{2}$$

The meaning of the symbols in the above expression is the following: A_{C} is the overall area of the heat exchange surface with fins, A_{i} is the area of the inner (smooth) wall of the tubes, A_{Z} is the area of fin surface, α_{i} is the heat transfer coefficient on the inner side of the tubes, α_{Z} is the heat transfer coefficient on the side with fins, δ_{w} is the thickness of tube walls, λ_{w} is the coefficient of the heat conductivity of the material of the fins and tubes and Ω is the efficiency of the fins.

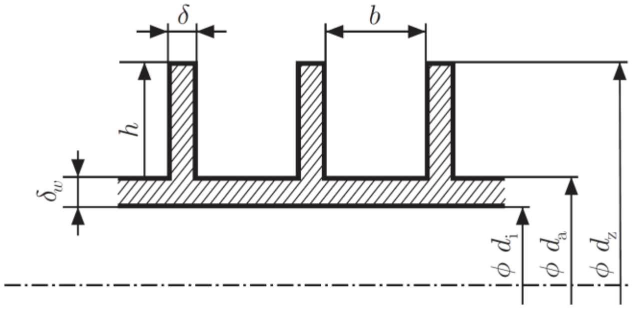

Figure 1: Axial cut through the tube with radial fins.

We consider *n* fins on a single tube with inner diameter of d_i, outer diameter of d_a and outer diameter of a fin d_Z. The distance between fins and the end fins and the walls of the heat exchanger is b, the thickness of the fin is δ and the height of the fin $h = (d_Z - d_a)/2$ (see Figure 1). The following relations hold:

$$A_Z = \pi n\left(d_Z^2 - d_a^2\right)/2 \tag{3}$$
$$A_C = \pi(n+1)bd_a + A_Z \tag{4}$$

If the length of the tube is L, then

$$A_i = \pi L d_i \tag{5}$$

The heat exchange area is calculated as a product mA_c, where *m* is the total number of tubes in the bundle and A_c is defined by equ. 4. The area A_Z does not include $\pi n d_Z \delta$ which is due to the definition of Ω - detailed analysis of this approach is laborious and is not practical to be introduced here. If water steam condenses inside the tubes, the heat transfer coefficient on this side is approximately equal to 10^4 W m^{-2} K^{-1}. The value of α_Z is determined from the knowledge of α_0, which is the heat transfer coefficient for the outer surface of a smooth tube with diameter of d_a under the same conditions. Because the converting relations between α_0 and α_Z are relatively complex, we show here graphical dependence of the ratio α_Z/α_0 on the ratio of *h*/*b* (see Figure 2).

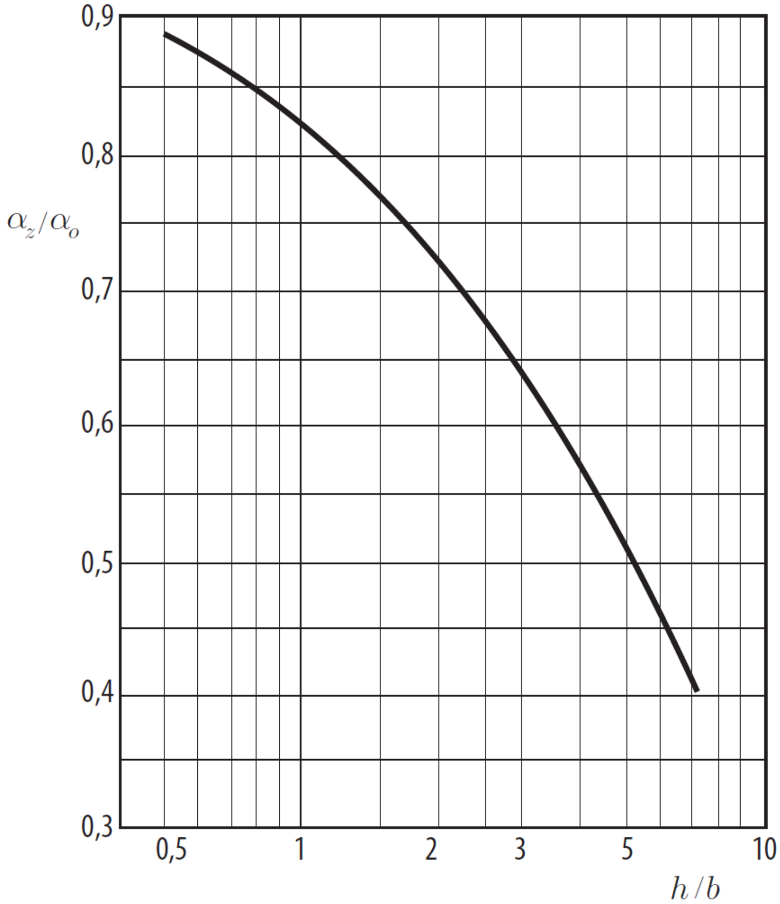

Figure 2: For calculating the heat transfer coefficient on tubes with fins.

One has to use the expressions for a bundle of tubes with a cross-flow to calculate the heat transfer coefficient α_0. In our case, the heated medium is always air and the exchanger is still the same. Therefore, the general relation can be simplified to

$$Nu = CRe^p \tag{6}$$

where *Nu* is the Nusselt number, $Nu = \alpha_0 d_a/\lambda$, λ is the coefficient of the heat conductivity of air, *Re* is the Reynolds number $Re = vd_a\rho/\eta$. The dynamic viscosity and the density of air are η and ρ, respectively, *v* is the mean air velocity at a minimal cross-section of the tube bundle *S*. The physical properties of air substituted into equ. 6 are taken at the inlet temperature of the air t_{Bi}. The values for C and p are dependent on the character of flow and are given in table 1.

Re	C	p
30 – 1000	0,48	0,50
$1 \cdot 10^3 - 2 \cdot 10^5$	0,24	0,60
$2 \cdot 10^5 - 1{,}6 \cdot 10^6$	0,14	0,84

Table1: The values of *C* and *p*. The correction for a small number of rows in the tube bundle is included in the constant *C.*

The efficiency of fins Ω which depends on α_z and on geometrical arrangement of the tube bundle can be read off from the Figure 3. Introduction of the efficiency of the fins into the relation 2 accounts for the fact that the temperature of the fins decreases away from the tube wall.

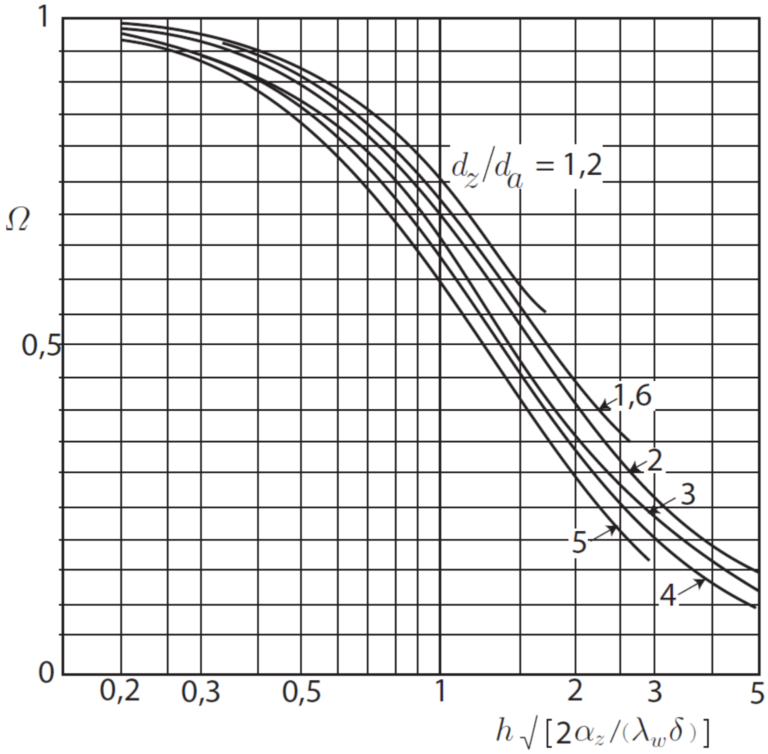

Figure 3: Efficiency of radial fins of rectangular cross-section.

2 Tasks

1) Derive the enthalpy balance of the heat exchanger and calculate the heat flow.
2) Calculate the value for the overall heat transfer coefficient k_Z from the measured values using equation 1.
3) Calculate the value of k_Z using equ. 2 and compare this value to the experimental one.
4) Discuss the effect of air velocity on experimental values of k_Z.

3 Description of the apparatus

The schematics of the apparatus is depicted in Figure 4. The dimensions of the apparatus needed for calculations are given in Table 2. The apparatus consists of the block 1 containing an in-built ventilator and a heat exchanger. The arrangement of the tubes with fins is depicted in Figure 5. The air is introduced on the suction side of the ventilator with a tube with cross-section of S_1. Turbine flow meter 2 is mounted on this tube and is connected to a programmable digital device. A thermometer 4 is placed before the entrance into the ventilator. A thermometer 5 is placed at the output tube of air. The flow rate of air is controlled with a flap valve 6. The heat exchanger is heated with water steam, a manometer 7, thermometer 8, regulation valve 9 and closing valve 11 are installed at the inlet of steam. The regulation valve is painted red, the closing one black. The closing valve cannot be used for smooth control of the steam flow rate but can be used for complete closing of the steam. The regulation valve (red) must not be closed completely because damage to the internal structures can occur. The condensate formed in the heat exchanger flows through the condensate pipe 10. The condensate is collected in a bucket 3 that is placed under the orifice of the pipe for the condensate. The ventilator is driven by an asynchronous motor and turns on with switch 12.

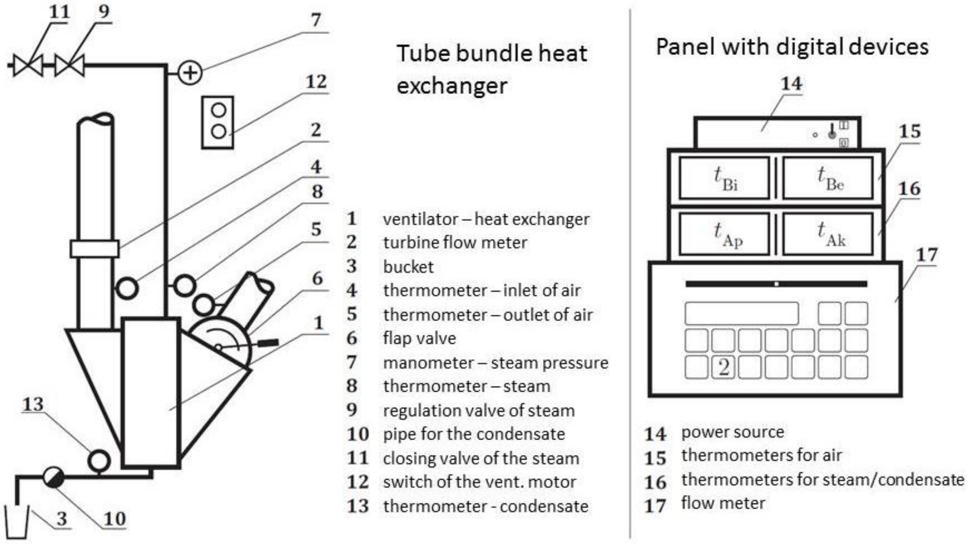

Figure 4: The schematics of the apparatus –side view

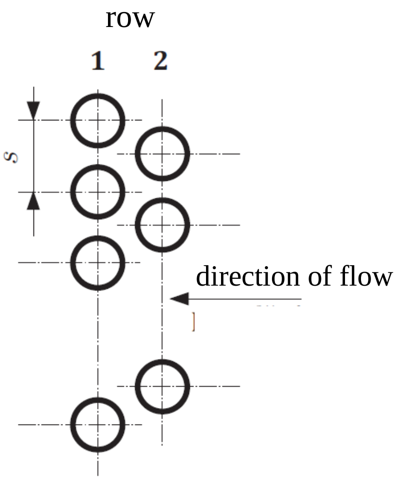

Figure 5: The arrangement of tubes in the heat exchanger.

Symbol	Name	Value
b	Distance between the fins	4 mm
d_a	Outer diameter of the tube in the bundle	17 mm
d_i	Inner diameter of the tube in the bundle	15 mm
d_z	Outer diameter of fins	32 mm
L	Length of the tubes	747 mm
m	Number of tubes in the bundle	45 total (23 – 1. row 22 – 2.row)
n	Number of fins on a single tube	165
s	Axial spacing of tubes in the bundle	36 mm
S	Minimal flow-through cross-sec. area of the bundle	$0{,}298\ \mathrm{m}^2$
S_1	Cross-section of the inlet pipe for air	$9{,}62 \cdot 10^{-2}\ \mathrm{m}^2$
δ	Thickness of the fins	0,5 mm
λ_w	Thermal conductivity of the tube material	$47\ \mathrm{W\ m^{-1}\ K^{-1}}$

Table 2: Geometrical parameters of the apparatus.

4 Procedure

The overall heat transfer coefficient k_Z is to be determined at two different air velocities. The procedure is the same in both cases.

1) The volumetric flow rate in $m^3\ h^{-1}$ is calculated from the given air velocity and the known cross-section of the inlet air tube (S_1). The result of this calculation is to be checked by an instructor or an assistant. Only then the students can continue with the work.
2) The power for the digital instruments is switched on with the switch 14. We wait until the value on the display of the flow meter 17 stabilizes. The electromotor of the ventilator is then turned on with a green push button 12 and the calculated flow rate is set with the flap valve 6. The main display of the flow meter does not show the actual flow rate, the number indicates the volume of the air that has flowed through since the apparatus was installed in the laboratory. This volume is given at 20°C and the pressure of 101325 Pa. To set the flow rate correctly, we will use a row display located above the main display. The row display shows the flow rate as a percentage when 100 % corresponds to 1700 $m^3\ h^{-1}$. The apparatus reaction to the movement of the flap valve is delayed by 5-10 seconds. One can check the set value of the flow rate by

pressing button “*Prutok”* (number 2) on the panel of the flow meter 17. The actual flow rate in $m^3\ h^{-1}$ is displayed for about 20 s. The value fluctuates significantly. This is caused by the high sensitivity of the flow meter. That is why the overall flowed-through amount of air is used for further calculations. This overall amount is obtained by integration which smooths out the fluctuations. This value is calculated continuously even if the display shows some other quantity. The value from the row display serves for keeping and adjusting the actual flow rate with the flap valve 6.

3) We will open valve 11 and set the required flow rate of steam by gradual opening of valve 9. The required flow rate of steam is given by the air output temperature measured with the thermometer 5. We read off the values of this temperature in 5-minute intervals on the panel 15. As soon as three subsequent readings of the temperature differ less than by 2 °C we will assume the steady state has been reached and will start the measurement. If we cannot reach the required output temperature, we will work with valve 9 completely open and write this fact in the report. When setting the output temperature, we have to take into account a non-negligible momentum of the device which is reflected in delayed reaction (a few minutes) to turning the valve. After reaching the steady state, we will write down the integral value of the flowed-through air shown on the main display of flow meter 17. We will place a bucket 3 under the tube with condensate.
4) When measuring, we record the following quantities in 5-minute intervals:
    a. time
    b. temperature of air at the inlet into the heat exchanger (thermometer 4, panel 15 left) - t_{Bi}
    c. temperature of air at the outlet from the heat exchanger (thermometer 5, panel 15 right) - t_{Be}
    d. temperature of the steam (thermometer 8, panel 16 left) – t_{Ap}
    e. temperature of the condensate (thermometer 13, panel 16 right) – t_{Ak}

    It is necessary to write down the value of overall amount of flowed-through air and pressure in the apparatus at the beginning and at the end of the steady state. The pressure is displayed for about 20 seconds after pressing the button “Tlak” on the flow meter 17. Further, we need to determine the overall amount of condensate obtained from the whole experiment by weighing of the condensate collected in the bucket. The manipulation with full bucket can be cumbersome (risk of condensate spill) so the use of two buckets shall be considered if one is filled quickly. The value of the instantaneous air flow rate is to be checked regularly by pressing the button 2 “Průtok” on the panel 17. If the flow rate is off the required value, we will use the flap valve 6 to adjust it.

5) The measurement is finished after collecting 10 values of each measured quantity (a. – e. see above).
6) If the measurement is completed for all required air flow rates, we will close completely the valve 11 (steam) and increase the flow rate of air to its maximum value with flap valve 6. We will turn off the ventilator after the temperature of the apparatus gets to about 40-50 °C (temperature of air at the output).

5 Safety precautions

1) It is not allowed to couple the apparatus to the inlet of steam without turning on the ventilator first.
2) When shutting down the apparatus, one must adhere to the instructions described in the point 6 of Procedures.

6 Processing the measured data

1) We will calculate the arithmetic mean for all measured quantities. We use the arithmetic means (notated with overbar) in all further calculations.
2) By subtracting the final and initial amount of flowed-through air and dividing with the time of measurement, we will get the air flow rate corresponding to conditions of 20°C and normal pressure. We will calculate the mass flow rate by multiplying the volumetric flow rate with air density calculated from the state equation for ideal gas at 20°C and normal pressure. The heat flow $\dot{Q}_\mathrm{B}$ will be determined from:

$$\dot{Q}_\mathrm{B} = \dot{m}_\mathrm{B}(\bar{t}_\mathrm{Be} - \bar{t}_\mathrm{Bi})\langle c_\mathrm{PB}\rangle \qquad (7)$$

where $\langle c_\mathrm{PB}\rangle$ is the specific heat capacity of air at its mean temperature: $\bar{t} = (\bar{t}_\mathrm{Be} - \bar{t}_\mathrm{Bi})/2$.

3) The heat flow $\dot{Q}_\mathrm{A}$ will be calculated from the collected amount of condensate according to:

$$\dot{Q}_\mathrm{A} = \dot{m}_\mathrm{A}(h_\mathrm{V} - h_\mathrm{K}) \qquad (8)$$

where h_V is the specific enthalpy of steam at the temperature measured with thermometer 8 and h_K is the specific enthalpy of the condensate at the mean temperature of the condensate $\bar{t}_\mathrm{Ak}$. We will use $\dot{Q}_\mathrm{B}$ for further calculations because the $\dot{Q}_\mathrm{A}$ is less precise due to the possible errors in $\dot{m}_\mathrm{A}$. The value of $\dot{m}_\mathrm{A}$ serves as a check for revealing errors in calculations or measurements.

4) We will calculate experimentally obtained value of k_Z from equ. 1 where

$$\Delta t_\mathrm{ls} = \frac{(\bar{t}_\mathrm{Be} - \bar{t}_\mathrm{Bi})}{\ln\frac{(\bar{t}_\mathrm{A} - \bar{t}_\mathrm{Bi})}{(\bar{t}_\mathrm{A} - \bar{t}_\mathrm{Be})}}. \qquad (9)$$

$\bar{t}_\mathrm{A}$ is the mean temperature of steam.

5) Following the instructions in the first section, we will calculate k_Z according to equ.2 and will express its relative deviation from the experimental value of k_Z as a percentage related to the experimental value. The averaged velocity of air flow, which is substituted into the *Re* number is calculated from the volumetric flow rate from point 2 that has to

be recalculated with respect to the actual conditions in the apparatus (actual pressure in the apparatus and the temperature at the inlet).

7 Symbols

A_c	overall surface area of a single tube with fins	m^2
A_i	surface area of the smooth side of the tube	m^2
A_Z	surface area of fins on one tube	m^2
b	the distance between fins	m
d_a	outer diameter of the tube	m
d_i	inner diameter of the tube	m
d_Z	outer diameter of the fins	m
h	height of a fin	m
k_Z	overall heat transfer coefficient for surface with fins	W m^{-2} K^{-1}
L	length of the tube	m
m	number of tubes in the bundle	
n	number of fins on a single tube	
S	minimum flow-through cross-sectional area of the tube bundle in the HE	m^2
S_1	cross-section area of inlet tube for air	m^2
v_1	mean air velocity in the suction tube	m s^{-1}
α_0	heat transfer coefficient on the outer surface of the smooth tube	W m^{-2} K^{-1}
α_i	heat transfer coefficient on the inner surface of the tube	W m^{-2} K^{-1}
α_Z	heat transfer coefficient on the side of the tube with fins	W m^{-2} K^{-1}
δ	thickness of the fin	m
Ω	efficiency of the fins	

8 Questions before the work

1) At how many air flow rates will you measure the overall heat transfer coefficient?
2) In what order will you switch on the devices?
3) When can you open the steam inlet into the heat exchanger?
4) How is the air flow rate set?
5) How can we double-check the correct flow rate of air?
6) What measured quantity determines the flow rate of the steam?
7) Where do you measure the temperature of the air entering and leaving the heat exchanger?
8) When can you start the measurement?

9) When is the steady state reached?
10) How to proceed in case you do not reach the required output temperature of air?
11) How do you measure the consumption of steam?
12) What physical quantities are to be measured? Show the placement of the sensors.