4 Pressure drop in packed beds

Miloslav Ludvík, Milan Jahoda, Martin Kohout

I Basic relations and definitions

The flow of the liquid or the gas through a static bed of the particulate solid is of interest in many chemical technology applications. Particulate assemblies can be spheres, cylinders, granulate material or special filling particles used in plants for mass transfer (adsorption, desorption and rectification columns). Ionex polymer as an example of spherical particles are used in production of demineralised water, particles with spherical or cylindrical shape are used as a catalytic materials in some types of reactors. In industrial processes the amount of produced gas or liquid (i.e. power output of the apparatus) is on one side limited by power output of the apparatus for their transport. On the other hand limitation is caused by pressure drop (hydraulic resistance of the apparatus). Knowledge of the dependence of the pressure drop on the liquid flow or on the other factors (porosity of a packed bed, particle size and shape, density and viscosity of the liquid etc.) is therefore very important for apparatus construction design and its operation.

Porosity of a packed bed is defined as

$$\varepsilon = \frac{V_\mathrm{f}}{V_\mathrm{B}} = \frac{V_\mathrm{B} - V_\mathrm{P}}{V_\mathrm{B}} \tag{4-1}$$

where V_B is volume of the packed bed, V_f is volume occupied by the liquid and V_P is volume of the particles in bed. Another characteristic of the packed bed is specific surface area of the packed bed

$$a = \frac{A}{V_\mathrm{B}} \tag{4-2}$$

where A is the surface area of the particles forming a packed bed with the volume V_B. The liquid flow is characterised by modified Reynolds number

$$Re_\mathrm{p} = \frac{\upsilon\, d_\mathrm{p,ek}\, \rho}{\eta} \tag{4-3}$$

where υ is superficial velocity calculated from the volume flow of the liquid $\dot{V}_\mathrm{f}$ and cross-sectional area of the tube S.

$$\upsilon = \frac{\dot{V}_\mathrm{f}}{S} \tag{4-4}$$

Equivalent diameter of the particle $d_\mathrm{p,ek}$ in Equation (4-3) is calculated as

$$d_\mathrm{p,ek} = \frac{6(1-\varepsilon)}{a} \tag{4-5}$$

Notice that the equivalent diameter of spherical particle is equal to its diameter.

The pressure drop in packed bed of randomly packed non-porous particles is often described by Ergun equation – equation where overall pressure drop per bed depth is equal to sum of the drops caused by viscous friction and by turbulent dissipative energy:

$$\frac{\Delta p_{\mathrm{dis}}}{h} = k_1 \frac{(1-\varepsilon)^2}{\varepsilon^3} \frac{\eta \upsilon}{d_{\mathrm{p}}^2} + k_2 \frac{1-\varepsilon}{\varepsilon^3} \frac{\rho \upsilon^2}{d_{\mathrm{p}}} \tag{4-6}$$

where k_1 , k_2 are empirical coefficients. Equation (4-6) can be transformed to the dimensionless form

$$f_{\mathrm{v}} = k_1 + k_2 \frac{Re_{\mathrm{p}}}{1 - \varepsilon} \tag{4-7}$$

where f_{v} is modified friction coefficient (proportion of the overall pressure drop and term defining energy drop in viscous area of the flow). Modified friction coefficient is defined as

$$f_{\mathrm{v}} = \frac{\Delta p_{\mathrm{dis}}}{h} \frac{d_{\mathrm{p}}^2}{\eta \upsilon} \frac{\varepsilon^3}{(1-\varepsilon)^2} \tag{4-8}$$

Empirical constants k_1, k_2 can be calculated by linear regression analysis of the dependence (4-7). This dependence is valid for all areas – laminar, transitional and turbulent areas of the liquid flow.

For packed bed where channels between the particles are connected (for example packed bed of filling particles used in apparatus for mass transfer – Raschig or Pall rings etc.) the description of the liquid flow is very complex. It is therefore impossible to use approximation of a parallel channels. In this case the pressure drop is described by empirical equation in following form:

$$\Delta p_{\mathrm{dis}} / h = k_1 F_{\mathrm{f}}^{k_2} \tag{4-9}$$

where k_1, k_2 are empirical constants, with $k_2 \to 2$ (as can be seen from definition of the depth drop in Bernoulli equation). In Equation (4-9) F_{f} is the intensity factor for the liquid defined as

$$F_{\mathrm{f}} = \upsilon \rho^{0,5} \tag{4-10}$$

where υ is the superficial velocity of the liquid.

II Objectives

1. Measure dependence of the pressure drop on the volume flow of the liquid for two tubes.
2. By regression analysis calculate constants k_1, k_2 from Equations (4-7) and (4-9).
3. Plot the measured dependencies graphically.

III Plant description

The scheme of the laboratory apparatus is shown in Fig. 4-1. Water from reservoir **15** is transported by the pump **14** through flowmeter **13** to the tubes with packed beds **1** to **3**. Pump is switched on by switch **11**. The value of the liquid flow in l s^{-1} is displayed on display **10**.

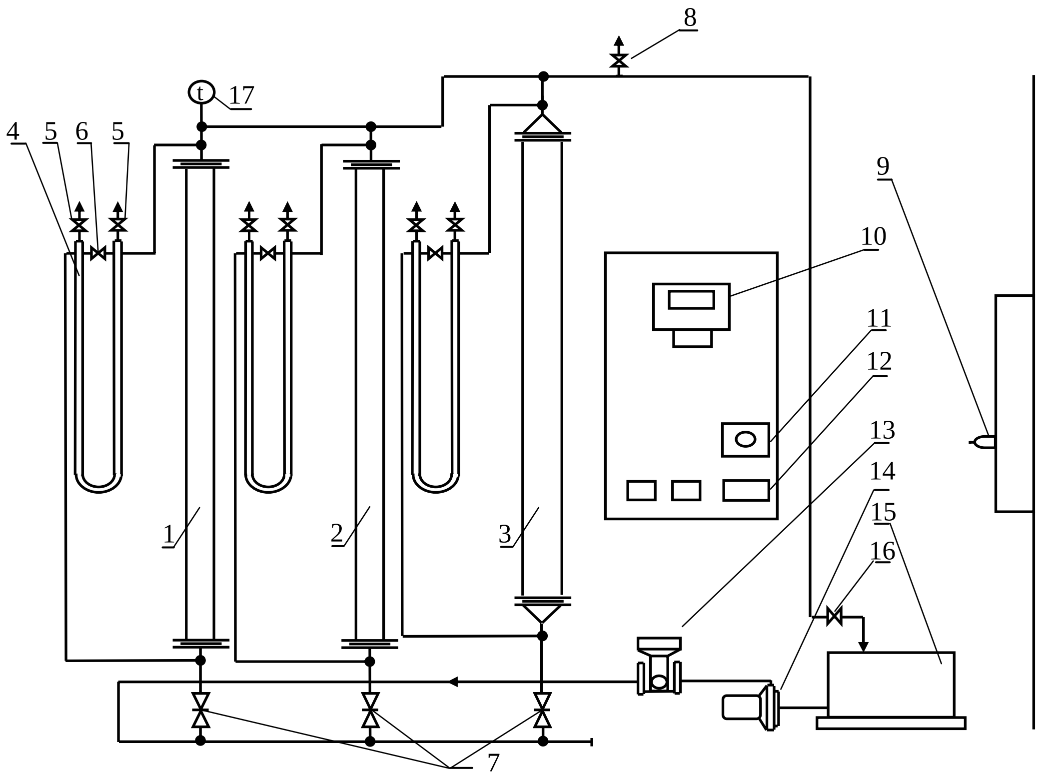

Fig. 4.1 Apparatus for the measurement of the pressure drop during the flow through a packed bed

1 to 3 tubes with packed bed of:
  1 - ceramic Raschig rings
  2 - glass spheres
  3 - plastic Pall rings
4 differential manometer for tube 1
5 manometer petcocks
6 shorting jumper (safety valve)
7 closing and governing valves
8 petcock of the apparatus
9 the electric power switch
10 display of the liquid flow
11 switch of the pump motor
12 wire of the pump motor
13 flowmeter
14 pump
15 water reservoir
16 closing valve
17 thermometer

The packed bed in the tube is restricted at the bottom and at the top by the perforated barriers. The water flow is regulated by governing valves **7**, which are also used for closing non-measured sections. Pressure drop arising from the water flow through the packed bed is measured by differential manometers connected to the corresponding tubes. Each manometer consists of one shorting jumper (safety valve) and two petcocks for deaerating of the manometer. Thermometer **17** is at the top left part of the apparatus. Closing the valve **16** on the returning pipeline to the reservoir can increase the pressure in the apparatus during deaerating of the manometers. This valve must be opened during the measurement. Electric power to the apparatus panel is switched on by switch **9** on the wall (right from the apparatus).

IV Work description

IV.1 Preparations

First, check if the closing valve **16** and shorting jumper (safety valves) of all manometers **6** are open. Governing valves **7** for each tube and petcocks **8** of all manometers must be closed. Switch on the switch **9** (the power supplement). It is necessary *to deaerate the tube and the manometer* before you start the measurement. Switch on the pump by switch **11**. By slow opening of the governing valve **7** before the measured tube reach its deaerating (bubbles of the air in the flow inside the packed bed will not be visible). Due to the safety valve is opened the level difference of the manometer is not significant. After the deaerating of the measured tube deaerate connected manometer. Governing valve **7** is opened, the liquid flows through the packed bed, manometer petcock are closed, safety valve is completely open and the level difference of the manometric liquid is not significant. Slowly open the petcocks and watch carefully the level difference. If there are rapid changes close the petcocks and call the assistant. During deaerating of the manometer part of the liquid runs out to the reservoir **15**. Deaerating is complete when bubbles of the air are not visible. After deaerating close petcocks, then close governing valve **7** and then close safety valve. The tube with packed bed and differential manometer are now ready for the measurement.

IV.2 Measurement of pressure drop in packed beds

Calculate the pressure drop from measured values of Δh from differential manometer. It is necessary to measure 20 values in range from 15 mm to maximal possible value on each manometer. Simultaneously write corresponding volume flow of the liquid (displayed on the flowmeter panel). The liquid flow is regulated by governing valve before the measured tube. Read the values after stabilising the fluctuations (after each flow change).

IV.3 End of the work

After the last measurement read the temperature of the water and write it to the laboratory protocol. Close all governing valves **7** and open the safety valve for measured tube. The measurement of the pressure drop in the second tube follow the same procedure as was described in Sections IV.1 a IV.2, i.e. it is necessary to deaerate the tube and the manometer before you start the measurement. After finishing the measurement switch of the pump (by switch **11**) and the power supplement (by switch **9**). Write the density of the manometric liquid to the laboratory protocol. After finishing final measurement please hand over the plant to assistant.

V Safety instructions

1. Increase the flow of the liquid carefully – the manometric liquid in differential manometers can run out very easily.
2. Always open safety valve during deaerating manometers (by opening petcocks).

VI Experimental data processing

VI.1 Packed bed with spherical particles

First, calculate the porosity and the specific surface of the packed bed for packed bed of spherical particles. Use the data in your laboratory protocol (diameter of the tube and the particles, number of particles). The volume of spheres is defined as

$$V_p = n \frac{\pi d_p^3}{6} \quad (4\text{-}11)$$

where n is the number of spheres. The surface area of the particles is calculated as

$$A = n \pi d_p^{\,2} \quad (4\text{-}12)$$

Combining Equations (4-11) and (4-12) with definitions (4-1), (4-2) and with definition of the volume of the packed bed $V_B = h \pi d^2 / 4$, one can obtain relations:

$$\varepsilon = 1 - \frac{2 n d_p^{\,3}}{3 d^2 h} \quad (4\text{-}13)$$

$$a = \frac{4 n d_p^{\,2}}{d^2 h} \quad (4\text{-}14)$$

Superficial velocity and Reynolds number can be calculated from the volume flow using Equation (4-4) and (4-3). From Equation (4-8) calculate modified friction coefficient and for acquired set of values $\left[\frac{Re_p}{1-\varepsilon}, f_v\right]$ calculate constants in Equation (4-7) by linear regression analysis (for example in Microsoft Excel). From measured values (reading of Δh on manometer) calculate values of the pressure drop by using Equation (4-15). Find carefully the value of the density of the manometric liquid ρ_m. Plot the regression curve together with experimentally measured values in graph.

$$\Delta p_{dis} = \Delta h \, (\rho_m - \rho) g \quad (4\text{-}15)$$

VI.2 Packed bed with non-spherical particles

From measured values of the volume flow calculate intensity factor using Equation (4-10). From Equation (4-15) assign the values of specific pressure drop for this independent variable and from Equation (4-9) calculate constants k_1, k_2 by regression analysis (using Microsoft Excel regression functions). Plot the regression curve together with experimentally measured values in graph.

VII List of symbols

a	specific surface of packed bed (per packed bed volume)	m^{-1}
$d_{p,ek}$	equivalent diameter of the particle	m
f_v	modified friction coefficient	
F_f	intensity factor	$m\, s^{-1} (kg\, m^{-3})^{0,5}$
h	bed depth	m
k_1, k_2	regression constants in Equations (4-7) a (4-9)	
n	number of particles	
Re_p	modified Reynolds number	

S cross-sectional area of the tube m^2

VIII Questions

1. What is porosity of the packed bed and how does it affect the pressure drop of packed bed?
2. How can be measured average superficial velocity of the liquid?
3. How can be measured pressure drop?
4. Which technological processes are modelled by laboratory apparatus? Show the examples!