9 Mixing

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I Fundamental relations and definitions

Mixing is a hydrodynamic process, in which different methods are used to bring about motion of particles in a mixed batch. Mixing serves in homogenization of mutually miscible liquids, suspension of granular material, dispersion, intensification of heat and mass transfer, etc.

Further we will be concerned with mixing of Newtonian liquids by mechanical impellers in a vessel. The action of an impeller on a mixed batch, whereby the impeller transfers some of its momentum to the liquid, results in the circulation of liquid in the vessel. In the case of rotational impellers with blades, the transfer of momentum results from the pressure on the liquid by the impeller blades, such that part of the liquid in front of the blade penetrates the surrounding liquid and another part moves in the direction of the rotating impeller. The so-called primary flow of liquid develops (i.e. liquid stream emanating from the rotor region of the impeller), which further transfers momentum to the surrounding liquid by turbulent and viscous friction. Just behind the blade, an underpressure develops, which causes suction of liquid from the vicinity of the impeller. Pumping and suction of liquid results in turbulent eddies around the impeller blades.
A typical arrangement of a mixing apparatus is shown in Fig. 9-1.

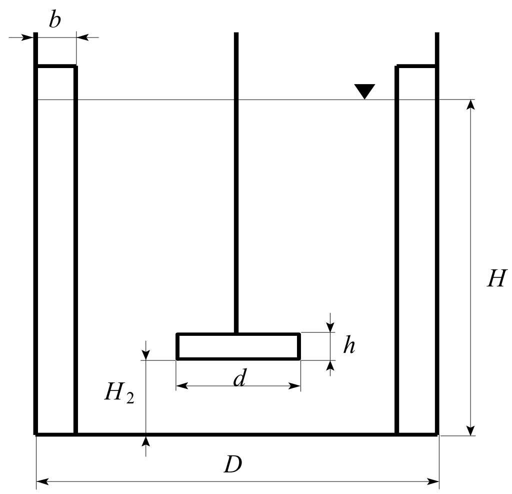

Fig. 9-1 Scheme of a mixing apparatus

A cylindrical vessel of diameter D is filled with the mixed liquid (batch) of density ρ and viscosity η to a height H from the bottom. The vessel can be equipped along the walls with vertical baffles which serve to suppress the formation of central vortices at high impeller speed n. the baffles are characterized by a width b and their number n_b. On the axis of the vessel, a rotational impeller of diameter d and height h is mounted at a height H_2 from the bottom. If the impeller has blades (most common case), it is further characterized by the number of blades and the angle of their inclination with the horizontal plane α – the pitch – (for vertical blades α = 90°).

One of the basic characteristics for the design of an impeller is the power draw. Elaboration of the Navier-Stokes Equations with the appropriate initial and boundary conditions by the similarity theories, it can be derived that the power draw of the impeller P_M is given by a basic equation between dimensionless criteria and geometrical simplexes for mixing

$$Po_{\mathrm{M}} = \mathrm{f}\left(Re_{\mathrm{M}}, Fr_{\mathrm{M}}, \Gamma_1, \Gamma_2, \ldots\right) \tag{9-1}$$

where Po_{M} is the power number for mixing

$$Po_{\mathrm{M}} = \frac{P_{\mathrm{M}}}{\rho\, n^3\, d^5} \tag{9-2}$$

Re_{M} is the Reynolds number for mixing

$$Re_{\mathrm{M}} = \frac{n\, d^2\, \rho}{\eta} \tag{9-3}$$

Fr_{M} is the Froude number for mixing

$$Fr_{\mathrm{M}} = \frac{n^2\, d}{g} \tag{9-4}$$

In the geometrical similarity simplexes Γ_{i}, complementing Eq. (9-1), are contained dimensions and indications according to Fig. 9-1 and table 9-1. The list of simplexes in table 9-1 can be simplified or further complemented in view of the given geometrical arrangement of the mixing equipment and the impeller type.

Table. 9-1. Definition of geometrical similarity simplexes

Simplex	Definition	Significance of symbol
Γ_1	D/d	b - width of baffles
Γ_2	H/d	D - inner diameter of vessel
Γ_3	H_2/d	d - impeller diameter
Γ_4	b/d	H - height of liquid batch
Γ_5	h/d	H_2 - height of the bottom end of impeller from vessel bottom
Γ_6	n_{L}	h - height of impeller
Γ_7	n_{b}	n_{L} - number of impeller blades
Γ_8	$\sin\alpha$	n_{b} - number of baffles in vessel
		α - angle of inclination of the impeller blades

The relation between measured quantities, (e.g. power dram P_{M}, impeller speed n etc.) can be suitably expresses as a relation between given dimensionless criteria and geometrical simplexes. In a case where we perform measurements in steady state under conditions in which vortices do not form (the gravitational acceleration g does not apply and hence the Froude number Fr_{M} too), the empirical relation Eq. (9-5) is assumed to hold

$$Po_{\mathrm{M}} = A\; Re_{\mathrm{M}}^{m0}\; \Gamma_1^{m1}\; \Gamma_2^{m2} \ldots \Gamma_8^{m8} \tag{9-5}$$

where the constant A and the exponents m_0, m_1, …, m_8 are usually different for laminar, transitional and turbulent flow regions.

Empirical results are often expressed graphically, as shown by Fig. 9-2 for basic types of impellers.

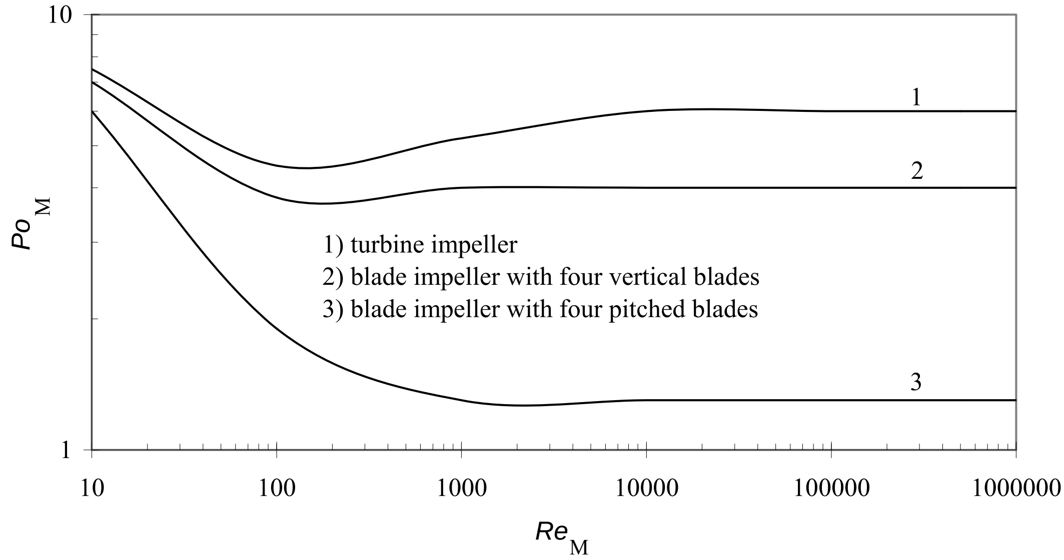

Fig. 9-2 Dependence of the power number on Reynolds number for different kinds of impellers in a baffled vessel (D/d = 3; H/d = 3; H_2/d = 1; n_b = 4)

From Fig. 9-2, it is apparent that in the turbulent region, occurring at $Re_M > 10^4$, the influence of Reynolds number does not apply ($m_0 = 0$). In such cases, if we study the influence of one of the geometrical similarities on the power draw (while maintaining constant the rest of the geometrical simplexes), it is possible to reduce Eq. (9-5) to

$$Po_M = A\Gamma_i^{m_i} \qquad \text{where } i \in \langle 1, 8 \rangle \qquad (9\text{-}6)$$

II Aim of the work

1. Assess the independence of Po_M on Re_M in the turbulent region for the batch
2. Determine the effect of the given change in the geometrical arrangement of the mixing equipment on the power draw of the impeller in the turbulent flow region. Determine the parameters A and m in Eq. (9-6).

III Description of the apparatus

The main parts of the mixing station are impeller, impeller drive, vessel, switchboard and measuring devices. A brief characteristic of the main parts (see Fig. 9-3):

A. Impeller

The impeller **2** is secured by a nut **4** at the bottom end of the vertical shaft **5**. The height of the impeller above the bottom is constant at 0.07m. The station is equipped with interchangeable blade and turbine impellers. An overview of impellers is given in table 9-2.

Table 9-2 Overview of the types of experimental impellers

Number	Type	Diameter	Blades	
		[m]	number	inclination
1	blade	0,15	2	90°
2	blade	0,15	2	45°
3	blade	0,15	4	90°
4	blade	0,15	4	45°
5	blade	0,15	4	60°
6	Rushton turbine	0,15	3	90°
7	Rushton turbine	0,15	4	90°
8	Rushton turbine	0,15	6	90°

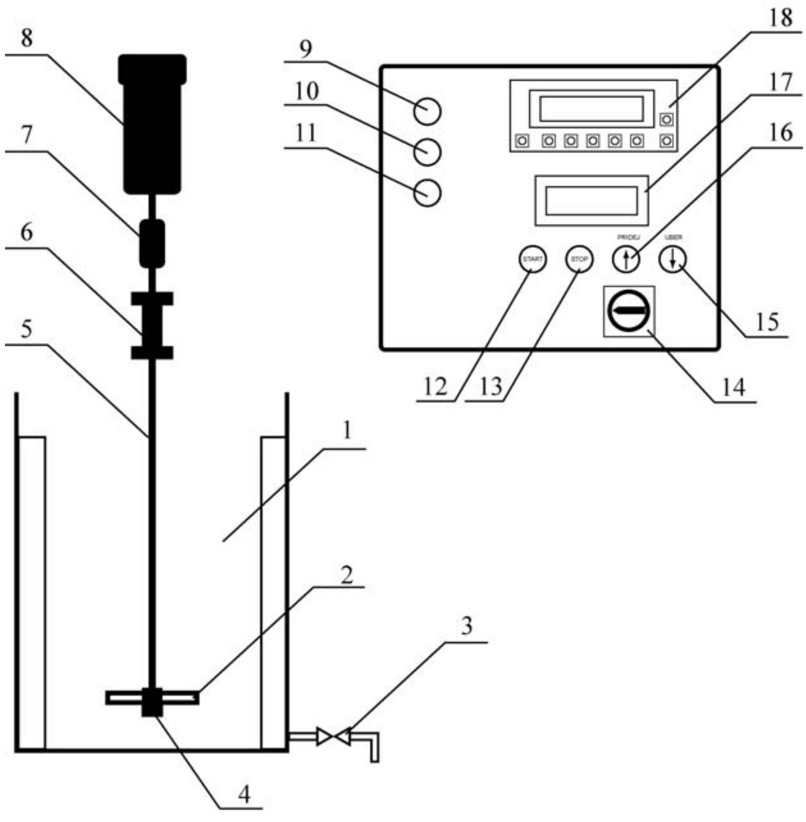

Fig. 9-3 Scheme of the apparatus

1 reservoir
2 impeller
3 emptying valve
4 securing nut for impeller
5 shaft
6 shaft bearing
7 dynamometer
8 electric motor with a cooling ventilator
9 control signal ZAPNUTO - OFF
10 control signal CHOD - RUNNING
11 control signal PORUCHA - FAULT
12 START button
13 STOP button
14 main switch
15 button for decreasing impeller speed (UBER - REDUCE)
16 button for increasing impeller speed (PŘIDEJ - INCREASE)
17 measurement of moment
18 impeller speed measurement

B. Driving mechanism

The shaft is driven via a counter gear by an electric motor **8**, which is cooled by a ventilator. The rotational speed of shaft can be changed using the buttons **15** and **16** on the switchboard.

C. Vessel

The vessel **1** is open and cylindrical with a flat bottom of inner diameter D = 0.6m and is mounted co-axially with the impeller shaft. On its walls are clamped four vertical baffles.

D. Switchboard and measuring devices

On the switchboard is the main switch of the whole apparatus **14**, a button for switching on the electric motor **12** (START), a button for switching off the electric motor **13** (STOP), control buttons for regulating the impeller speed **15** (lowering the speed) and **16** (increasing the speed), a display from the moment measuring device **17** and a depiction of the shaft speed **18**. Furthermore, signal controls are placed here. The signal control **9** (ZAPNUTO-SWITCHED ON) lights up after switching on the main switch, signal control **10** (CHOD-RUNNING) lightens up after switching on the button **12** (START) and switches off after pressing the button **13** (STOP) and the signal control **11** (PORUCHA-FAULT) lights up when there is a problem, e.g. when the shaft is blocked or overheating the motor.

IV Experimental procedure

IV.1 Preparation

a) We switch on the main switch of the apparatus **14**, the signal control **9** lights up.
b) With the aid of an instructor, we mount the prescribed impeller type on the shaft **5**, which we secure with the nut **4**.
c) We fill in the vessel with water at a height which is equal to the vessel diameter (i.e. 0.45m) and we measure its temperature.
d) From Eq. (9-3), we calculate the impeller speed for the value $Re_M = 10^5$, i.e. the value at which turbulent flow of the batch in the vessel reliably exists. The first experimental point will be measured at the impeller speed equal to 50% of this calculated impeller speed, the next point at 100%. We set these values in the first two rows of the protocol form for the given impeller.
e) We find the highest impeller speed, at which we will measure the power draw for the given impeller. We switch on the electric motor by switch **12** (START) and by repeatedly pressing the button **16** (PŘIDEJ-INCREASE), we find the maximum frequency, at which the total moment 5Nm is reached (depicted on the display **17**). The impeller speed can be changed while the apparatus is running. We record the maximum impeller speed on the last row in the protocol form, for the given impeller.

IV.2 Measurements

We distribute the impeller speeds between the initial (2nd row in the protocol form) and the maximum value, for the remainder of the experimental values, for which there spaces in the form. We should appropriately round off the values, such that the setting of the selected impeller speed is easier. The distribution of the impeller speeds should be done for each impeller individually!

During the experiment, we gradually set the pre-selected impeller speed using buttons **15** or **16** and we read off the moment measured by the dynamometer. We write down the moment into the column (*moment-total* - *moment-celkový*). After measuring the total moment at all the impeller speeds, we decrease the impeller speed to zero using button **15** (REDUCE-UBER) and we switch off the electric motor using switch **13** (STOP). We then remove the impeller from the shaft and fix back the securing nut to the shaft. We repeat the measurements of the moments at the same speeds and we write down the measured values into the protocol column *moment-shaft* (*moment-hřídel*).

IV.3 Termination of the experiment

We set the impeller speed to zero, switch off the electric motor and empty the vessel. The main switch should be left on, because it is necessary to wait till the ventilator cools the electric motor.

V Safety precautions

1. When mounting the impeller, the electric motor should be switched off by button **13** (STOP) on the switchboard, the control signal **10** (RUNNING-CHOD) is off and the impeller speed set to zero.
2. Before switching on the electric motor, the impeller speed should be set to zero.
3. The impeller speed should be regulated slowly.
4. The maximum impeller speed is limited by the moment 5Nm.

VI Processing of measured data

We first calculate the moment of the impeller as the difference between the total moment and the moment measured with the demounted impeller (moment-shaft). From the calculated moment we then calculate the impeller power draw P_M according to the equation

$$P_M = 2\pi M n \tag{9-7}$$

If we use the basic SI units, i.e. we install the moment M in Nm and the impeller speed n in s^{-1} we obtain the impeller power draw P_M in Watts. Measured values of the impeller speed and the impeller power draw serve in calculating the criteria Re_M and Po_M.

The power number Po_M is calculated from the definition relation (9-2). All quantities should be installed in SI units. For values of Po_M measured at $Re_M > 10^5$ and for each impeller individually, we calculate the mean value $\overline{Po}_M$, the sample standard deviation $s_{\overline{Po}_M}$ and we then evaluate the confidence interval.

From the assignment, it is apparent which parameter effect was being studied. This parameter determines one value of the relevant simplex Γ_i for each given impeller. For each impeller we have likewise evaluated the mean value of the power number. We have therefore two twin sets of values (Γ_{i1}, $\overline{Po}_{M1}$) and (Γ_{i2}, $\overline{Po}_{M2}$). Substituting these twin sets of values into Eq. (9-6) and solving the resultant system of two equations, we obtain the sought after parameters A and m_i. We then use the values of these coefficients for cross-checking by reverse substitution into Eq. (9-6).

In the protocol we further attach a graph of the dependence of power number on Reynolds number for both impellers (on the same graph).

VII Symbols

A	a constant in the power number dependency Eq. (9-5)	
b	width of baffles	m
D	inner diameter of the vessel	m
d	impeller diameter	m
H	liquid height above vessel bottom	m
H_2	height of the bottom end of the impeller from vessel bottom	m
h	height of the impeller blades	m
M	moment	Nm
m	exponent in the power number dependency	
n	impeller speed	s^{-1}
n_L	number of impeller blades	
n_b	number of baffles in vessel	
P	power draw	W
Po	power number from Eq. (9-2)	
Re	Reynolds number from Eq. (9-3)	
α	impeller blade angle of inclination	
η	dynamic viscosity of batch	Pa s
ρ	batch density	kg m^{-3}
Γ	geometrical similarity simplex	

VIII Control questions

1. What is the purpose of the mixing process?
2. What is the main function of the baffles?
3. What is the purpose of batch homogenization?
4. Explain the concept “geometrical similarity between mixing equipment”

5. How does batch circulation develop in a mechanically stirred vessel?