Pressure losses in fluid flow through straight pipes and fittings

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1 Basic relations and definitions

The description of fluid flow in conduits is based on (i) the continuity equation and (ii) the Bernoulli equation. The former expresses the conservation of mass, the later the conservation of mechanical energy.

The continuity equation written for a single pipe with steady fluid flow reads $\dot{m}_1 = \dot{m}_2$, where $\dot{m}$ is the mass flow rate and the indexes 1 and 2 denote the inlet and outlet cross-sections of the pipe, respectively. The mass flow rate of fluids in pipes can be expressed as a product of the fluid velocity *v*, the cross-section area of the pipe *S* and the fluid density

$$\dot{m} = \rho v S \tag{1}$$

Substituting this expression into the expression above, we arrive at:

$$\rho_1 v_1 S_1 = \rho_2 v_2 S_2 \tag{2}$$

that can be further simplified for a case of constant density to:

$$v_1 S_1 = v_2 S_2 . \tag{3}$$

Real fluid loses part of its mechanical energy by friction and by formation of eddies. The amount of energy lost in dissipation is often expressed in the form of equivalent pressure difference, so called pressure loss. This pressure loss occurs in fluid flow through straight pipes and through fittings and shaped pipes (local resistances). In this project we will deal with pressures losses in straight pipes and fittings. We will start with Bernoulli equation in the form of

$$\frac{p_1}{\rho} + \frac{v_1^2}{2} + h_1 g = \frac{p_2}{\rho} + \frac{v_2^2}{2} + h_2 g + e_{dis1,2} \tag{4}$$

where indexes 1 and 2 denote the cross-sections of the pipe at the inlet and the outlet, respectively, *p* is the pressure, *v* is the velocity, *h* is the geometrical height related to the cross-sections 1 and 2, ρ is the density of the flowing fluid, *g* is the gravitational acceleration, and $e_{\mathrm{dis1,2}}$ is the specific dissipative energy between cross-sections 1 and 2. The loss in pressure is related to loss in energy through

$$\Delta p_{\mathrm{dis1,2}} = \rho e_{\mathrm{dis1,2}}. \tag{5}$$

The loss of energy in/on fittings is usually characterized by loss coefficient ζ that is defined by

$$e_{\mathrm{dis}1,2} = \zeta \frac{v^2}{2}. \qquad (6)$$

The loss of energy in straight pipes is calculated with the use of friction factor λ introduced by equation

$$e_{\mathrm{dis}1,2} = \lambda \left(\frac{l}{d}\right) \frac{v^2}{2}, \qquad (7)$$

where l is the length of the pipe and d is its inner diameter. When calculating λ and ζ from experimental data, we start from the Bernoulli equation expressing the change in pressure $\Delta p = p_1 - p_2$

$$\Delta p = \frac{v_2^2 - v_1^2}{2} \rho + (h_2 - h_1)\rho g + e_{\mathrm{dis}1,2}\rho. \qquad (8)$$

When the geometrical heights of chosen cross-sections are same, i. e. $h_1 = h_2$, the equation can be simplified to:

$$\Delta p = \frac{v_2^2 - v_1^2}{2} \rho + e_{\mathrm{dis}1,2}\rho. \qquad (9)$$

If the cross-sectional areas are same, then the continuity equation predicts equal velocities v_1 and v_2 and the previous relation can be further simplified to:

$$\Delta p = e_{\mathrm{dis}1,2}\rho = \Delta p_{\mathrm{dis}1,2}. \qquad (10)$$

The pressure difference will be measured with a differential manometer. The relation between the pressure difference and the difference in levels of the working liquid in the manometer can be expressed as:

$$\Delta p = \Delta h(\rho_{\mathrm{m}} - \rho) g \qquad (11)$$

where ρ_{m} is the density of the working liquid. By combining the expressions (6) and (11), and (7) and (11), one can arrive at equations for calculating the loss coefficient and friction factor, respectively

$$\zeta = \frac{2\Delta p}{(\rho v^2)}, \qquad (12)$$

$$\lambda = \frac{2\Delta p d}{(l \rho v^2)}. \qquad (13)$$

We can determine the velocity of the flow from the volumetric flow rate and the cross-sectional area of the pipe.

The value of the friction factor λ (in general even the value of ζ) depends on the density and viscosity of the flowing fluid, the velocity of flow, the characteristic length dimension of the system (inner diameter for a circular pipe, for fittings it is the inner diameter of the pipe to which the fittings are mounted) and the roughness of the pipe. The similarity theory shows that the dependence on the density and viscosity of the flowing fluid, velocity of flow and characteristic length dimension can be expressed as a dependence on a single variable – Reynold number that is defined by:

$$Re = \frac{vd\rho}{\eta} \qquad (14)$$

for a pipe with circular cross-section.

The dependence of λ on *Re* was obtained experimentally (the same experiments you are about to conduct in the laboratory only the extent was larger). λ depends only on *Re* number up to its value of 2300. This region of flow is called laminar flow. The laminar flow is characterized by parallel layers of fluid sliding one over another. The momentum transfer in the direction perpendicular to the flow occurs on the molecular level only (viscosity). The flow regime with $Re > 10^4$ is called turbulent and is characterized by very intensive momentum transfer in the direction perpendicular to the flow through macroscopic turbulent eddies (increase in dissipation of energy). The formation of eddies enhances the increase in velocities of flow and sudden changes of flow direction. Sudden changes of flow directions occur e. g. in flow through fittings or in flow around fine unevenness on pipe walls whose mean height ε is called absolute surface roughness and its values are tabulated for various materials. Laminar boundary layer localized at the pipe walls is preserved (due to fluid having smaller velocity at the pipe walls) even for conditions when the flow is turbulent in the axis of the pipe. The thickness of the laminar boundary layer decreases with increasing *Re* number. The protrusions on the pipe walls can penetrate through the boundary layer and start to play a role in increasing the turbulence. λ does not depend on *Re* and is function of the relative surface roughness. It is apparent from the theoretical analysis why values of ζ are given in tables as constants independent on *Re*. One assumes that very strong turbulence develops in fittings. The experimental arrangement in the laboratory allows to verify this assumption.

2 Goals

1. Determine the friction factor for a chosen straight pipe and loss coefficients for given fittings.
2. Depict graphically the dependence of friction factor on *Re* in the measured range of conditions.
3. Determine the average value of loss coefficients measured at different flow rates.

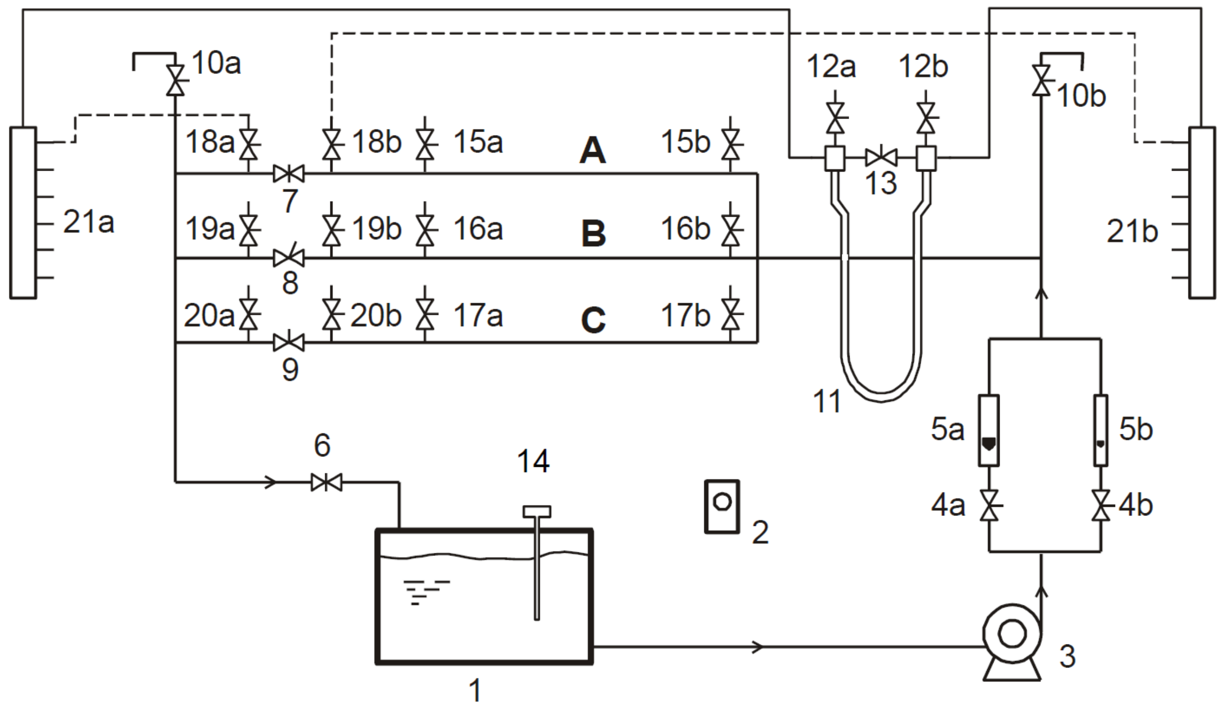

Figure 1: Schematics of the apparatus:

1 – reservoir; **2** – switch for the pump; **3** – pump; **4** – valves; **5** – rotameters; **6** – valve; **7** – valve, **8** – valve with skewed stem; **9** – valve with perpendicular stem; **10a,b** – valves for degassing the apparatus, **11** – differential manometer; **12** – valves for degassing the manometer; **13** – shortcutting stopcock of the manometer; **14** - thermometer; **15a,b** – valves for connecting the manometer to the straight pipe A; **16a,b** – valves for connecting the manometer to the straight pipe B; **17a,b** – valves for connecting the manometer to the straight pipe C; **18a,b** – valves for connecting the manometer to the fitting 7; **19a,b** – valves for connecting the manometer to the fitting 8; **20a,b** – valves for connecting the manometer to the fitting 9; **21a,b** – connecting modules;

3 Description of the apparatus

The apparatus is schematically shown in Figure 1. A centrifugal pump 3 is used to pump water from a storage reservoir 1 through one of the valves 4 into one the rotameters 5. The water is further lead into a manifold with inlets into three pipes A, B, C. Each pipe has a straight part and one fitting (7, 8, 9) installed on it. The individual measured parts of the pipe system are permanently connected to the manometer 11 through connecting modules 21 with connecting hoses. The dashed lines in the Figure 1 show the connection of the fitting 7 to the connecting modules 21. The other parts of the fluidic system are connected to the manometer in a similar way. When measuring the pressure drop on a straight part of the pipe, one opens the corresponding valves 15-20, **all others are closed**.

The differential manometer 11 for measuring the pressure difference on individual parts of the fluidic system is made of a glass U-tube filled with a working liquid immiscible with water.

The manometer is equipped with degassing valves 12 and one stopcock 13 for shortcutting the manometer. The valves 10 and 11 serve for degassing the apparatus when the apparatus is being filled with water. The valve 6 adjusts the volumetric flow rate of water through the apparatus.

4 Procedure

4.1 The preparation of the apparatus for measurement

1. We will check the amount of water in the storage reservoir 1. If the amount is low, we will add distilled water.

2. We will open fittings 7, 8, 9, and the degassing valves 10. Both valves 4 are open by two turns and the valve 6 is completely closed. We will check that all valves 15-20 on outlets to connecting modules for differential manometer are closed. We will start the pump with switch 2 and wait until water without bubbles flows back into the storage reservoir from hoses connected to the degassing valves 10. All bubbles have been removed from the apparatus. We will turn off the pump and close the valves 10.

3. We will close two of the three fittings 7, 8, 9 on the tubes that will not be used for the measurement. The fitting on the measured tube has to be fully open (e. g. we will measure on the tube A, fitting 7 is fully open, 8, 9 are closed). At the same time the correct valves from 15-20 will be open by 2-3 turns.

4. The manometer has to be degassed before the actual measurement too because the presence of bubbles can cause significant errors in measuring the pressure difference. We will open the valve 4b (leading into the smaller rotameter), 4a will be fully closed. We will open valve 6 by two turns. Then, we will fully open the shortcutting stopcock 13, start the pump and slowly open one of the degassing valves 12. The working fluid can be easily expelled from the manometer!! This is undesirable since it is toxic and expensive. If the liquid flowing out of one arms of the manometer is devoid of bubbles, the arm is degassed and we can proceed to degas the second arm. After degassing, we carefully close the shortcutting stopcock 13. The manometer is now ready for the measurement. Correct function of the manometer can be checked when both valves 4 are completely closed. At zero flowrate the pressure difference has to be equal to zero.

5. We will start the measurement by finding the working range for each measured fluidic component. The minimum flow rate is given either by the minimum measurable flow rate of the smaller rotameter or the minimum measurable pressure drop. The minimum flow rate is that one, at which the pressure drop is measurable. One can find the maximum flow rate by turning on the pump and slowly opening the valve 4a of the larger rotameter. At the same time we are opening the valve 6 and observe the rotameter and manometer. The maximum flow rate is either limited by the pump performance or the range of the larger rotameter is exceeded. Finding the working range has to be repeated for each of the measured fluidic component.

4.2 Measurement

We will write down the temperature shown on the thermometer 14 before starting the measurement. We will divide the difference in the maximum and minimum flow rate obtained in the point 5 of the previous paragraph into a number of same parts so that all rows in the handed form are filled out. We set the flow rate with valve 6 and measure the pressure drop, the obtained data are written into the form. We use valve 4 at low flow rates.

4.3 Completing the work

We will measure the temperature of water after the last fluidic part is measured. After finishing all required measurements we will close both valves 4 and turn off the pump with a switch 2 and close the valve 6.

5 Safety measures

1. We do not climb on the apparatus, it is not built for it.
2. We avoid touching the pump when running.
3. We set the required flow rate slowly so that the apparatus is not exposed to beats and sudden changes in pressure.

6 Data processing

The individual columns in the form will be calculated in the following way:

1. The flow rate from a calibration equation displayed at the apparatus.
2. The velocity from the flow rate and equation 1 ($\dot{m}/\rho$ equals the flow rate). Required dimensions of the apparatus are also displayed at the apparatus. The velocity for fittings will be calculated from the cross-section of the pipe these fittings are mounted to.
3. The pressure drop will be calculated from the data measured on the manometer and equation 11. The density of the working liquid is displayed at the apparatus.
4. *Re* number from equation 14. The viscosity and density at mean temperature will be read off from the tables.
5. Friction factor for straight pipes from equation 13.
6. Loss coefficients for fittings from equation 12.

The report is to contain a graph showing the dependence of the friction factor on the Reynolds number. The plot is in semi-logarithmic coordinates (logarithmic coordinates for *Re*). Note: Logarithmic coordinate on x axis does not mean to plot logarithm of the *Re* number. If *Re* = 10000, the plotted value is 10000 not 4. y axis is in decimal coordinates.

Calibration equation of the rotameter and other data displayed at the apparatus have to be rewritten during the measurement. The relation for calculating the pressure drop from the manometer requires substitution of all quantities in SI units.

7 Symbols

d	inner diameter of the pipe	m
e_{dis}	specific loss energy	m^2s^{-2}
g	gravitational acceleration	$m\ s^{-2}$
h	geometrical height of the pipe	m
l	length of the pipe	m
p	pressure in the pipe	Pa
Re	Reynolds number	
S	flow-through cross-sectional area	m^2
v	velocity of the fluid	ms^{-1}
$\dot{V}$	volumetric flow rate	m^3s^{-1}
Δh	the difference in levels of the working fluid	m
Δp	the difference in pressures, here pressure drop	Pa
η	dynamic viscosity	Pa s
λ	friction factor	
ρ	density of the flowing liquid	$kg\ m^{-3}$
ρ_m	density of the working fluid in the manometer	$kg\ m^{-3}$
ζ	loss coefficient	

8 Check questions

1. What is the goal of the work, what quantities are you going to set and measure?
2. What are you going to do before the measurement?
3. How are you going to proceed during the measurement?
4. How and when are you going to degas the apparatus?
5. How and when are you going to degas the manometer?
6. How are you going to make sure the working fluid of the manometer will not escape the manometer?
7. Can you touch valves when liquid flows through them? Can you start a centrifugal pump wen the valves at the discharge are closed?
8. Can you measure with both large and small rotameter at the same time?
9. How are you going to proceed in reading off the value from the U manometer? How can you test that the manometer has been degassed?
10. How much are you going to open or close the measured and non-measured fittings?