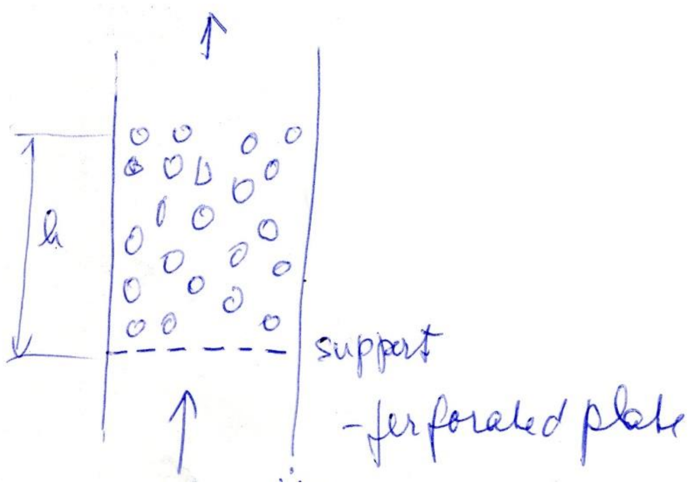
Fluidization

Fluidized bed is a suspension of solid particles in a fluid that is flowing against the direction of gravitational force. Mass and heat exchange is very intense → catalytic reactions, combustion, also drying. Fluidized bed behaves as fluid → transport of particulate material. Also, fluidized bed can be used to separate particles by their size or density.



- **voidage**:

$$\varepsilon = \frac{volume\ of\ fluid\ in\ the\ fluidized\ bed}{volume\ of\ the\ fluidized\ bed} = \frac{V_f}{V} \quad \text{(it is a volume fraction)}$$

$$1 - \varepsilon = \frac{V_s}{V} = \frac{volume\ of\ the\ solids}{volume\ of\ the\ fluidized\ bed}$$

$v = \frac{\dot{V}_f}{S}$ … **superficial velocity** of the fluid

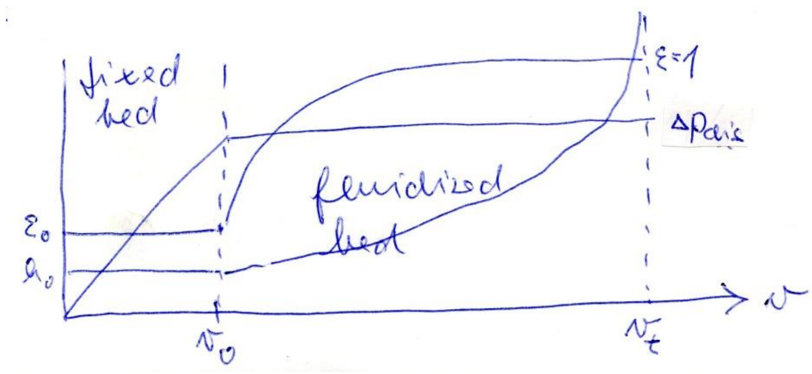
S … cross-sectional area

If the superficial velocity v is increased from zero value, then at first the particles are sitting on the supporting plate and do not move → **fixed bed**.

Above certain threshold value v_0, the fixed bed becomes a **fluidized bed** → particles start to move about and the height h increases.

For each $v > v_0$ there is a definite value of h. Voidage and height are constant for the fixed bed but increase with v for the fluidized bed.

However, pressure drop Δp_{dis} of the bed grows with v for the fixed be and remains constant for the fluidized bed.



As the voidage ε approaches 1, the particles become isolated, $h \to \infty$ and the conditions are equivalent to a freely falling isolated particle → settling at terminal velocity v_t. The pressure drop Δp_{dis} is reflecting the drag force, which increases with in the fixed bed because the forces acting on the bed are unbalanced, unlike in the fluidized bed where the forces are balanced. A more detailed description of forces is as follows.

Three forces act on the bed (positive direction is upward)

- **gravitational** (external): $F_g = -\rho_s g V_s$
- **buoyancy** (internal, pressure related): $F_{Ar} = \rho_f g V_s$
- **drag force** (internal; shear stress-related): $F_R = \Delta p_{dis} S$

For the fixed bed F_g dominates over $F_{Ar} + F_R$; F_R increases with v and at $v = v_0$ all three forces are balanced (steady state):

$$F_g + F_{Ar} + F_R = 0$$

thus

$$\Delta p_{dis} = \frac{(\rho_s - \rho_f) g V_s}{S} = \frac{(\rho_s - \rho_f) g V (1-\varepsilon)}{S} = (\rho_s - \rho_f) g h (1-\varepsilon) = const$$

This balance holds for any v in the range from v_0 to v_t because the height of the fluidized bed readjust upon increase in v so that the steady state is maintained. The formula for pressure drop explains why Δp_{dis} does not depend on v in the fluidized bed.

Determination of the minimum fluidization velocity v_0:

This situation is simultaneously fixed and fluidized bed and so pressure drop calculated from formulae for each case should be equal:

$$\Delta p_{dis} = \underbrace{\frac{3}{4} \lambda_{fix} \frac{h_0}{d_p} \frac{1-\varepsilon_0}{\varepsilon_0^3} v_0^2 \rho_f}_{fixed\ bed} = \underbrace{(\rho_s - \rho_f) g h_0 (1-\varepsilon_0)}_{fluidized\ bed}$$

where spherical particles of diameter d_p are considered and the friction factor for fixed bed is given by **Ergun equation**:

$$\lambda_{fix} = \frac{133}{\mathrm{Re}_{fix}} + 2.33 \ \text{ and } \ \mathrm{Re}_{fix} = \frac{2}{3(1-\varepsilon_0)} \underbrace{\frac{d_p v_0}{\nu_f}}_{\mathrm{Re}_0}$$

On introducing **Archimedes number**

$$Ar = \frac{g d_p^3 (\rho_s - \rho_f)}{\nu_f^2 \rho_f} \left(= \frac{g d_p^3 (\rho_s - \rho_f) \rho_f}{\eta_f^2} \right)$$

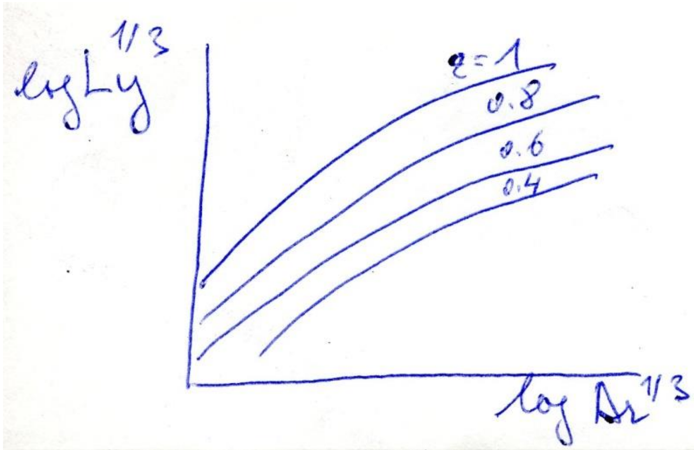
the condition for finding v_0 is put into dimensionless form:

$$Ar = 150 \frac{1-\varepsilon_0}{\varepsilon_0^3} \mathrm{Re}_0 + \frac{1.75}{\varepsilon_0^3} \mathrm{Re}_0^2$$

Expansion of the fluidized bed is conveniently described by introducing the **Lyaschenko number**

$$\mathrm{Ly} = \frac{\mathrm{Re}^3}{\mathrm{Ar}} = \frac{v^3}{g \nu_f} \frac{\rho_f}{\rho_s - \rho_f}$$

as a function $\mathrm{Ly}^{1/3} = f\left(\mathrm{Ar}^{1/3};\ \varepsilon\right)$ → empirical in the form of a graph



Notice that $\mathrm{Ly}^{1/3} \sim v$ and $\mathrm{Ar}^{1/3} \sim d_p$. Also notice that Ly does not depend on d_p and Ar does not depend on v.

Types of fluidized beds:

- **particulate fluidization** – voidage does not depend on time (but may depend on height)
- **aggregating** or **bubbling fluidization** – depends on time as well as position in the bed (often irregular and complex)

The quantity

$$\frac{\rho_s - \rho_f}{\rho_f} \text{ (=Archimedes simplex)}$$

helps to discern between the two cases, typically:

1) $\frac{\rho_s - \rho_f}{\rho_f} < 10$ particulate (mostly liquids)
2) $\frac{\rho_s - \rho_f}{\rho_f} > 10$ aggregating (gases)

Remark: Terminal velocity v_t is calculated from the balance of forces acting on an isolated spherical particles with the result:

$$(\rho_s - \rho_f) g d_p = \frac{3}{4} \zeta_d v_t^2; \ \ \zeta_d \text{ ... drag coefficient}$$

or

$$\mathrm{Ar} = \frac{3}{4} \zeta_d \mathrm{Re}^2; \quad \mathrm{Re} = \frac{v_t d_p}{\nu_f} = \frac{v_t d_p \rho_f}{\eta_f}$$

Supporting material for fluidization

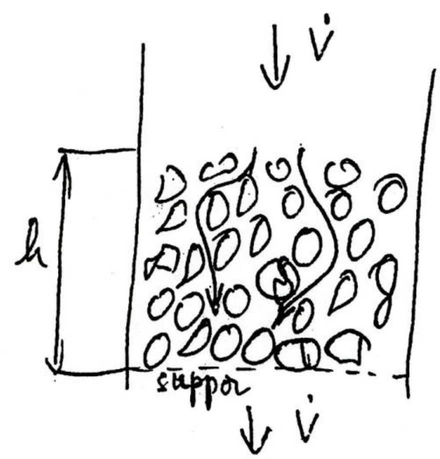
Fluid flow through a bed of solids

Application:

- filtration
- heterogeneous reactors
- adsorption and chromatography

Aim is to calculate pressure drop. Bed of solid particles is characterized by:

- random arrangement of particles
- irregular channels within the bed



Properties:

cross-sectional area: S

volume of fluid: V_f

volume of particles: V_S

volume of the bed: $V_B = hS = V_f + V_S$

surface of particles: A_S

void fraction (porosity):
$\varepsilon = \frac{V_f}{V_S}$ (= volume fraction of fluid)

volume fraction of particles: $\frac{V_S}{V_B} = 1 - \varepsilon$

superficial velocity: $v = \frac{\dot{V}}{S}$

interstitial velocity: $v_\varepsilon = \frac{\dot{V}}{\varepsilon S} = \frac{v}{\varepsilon}$

specific surface: $a = \frac{A_S}{V_B} = (1-\varepsilon)\frac{A_S}{V_S}$,

for spheres $a = (1-\varepsilon)\frac{6}{d_p}$

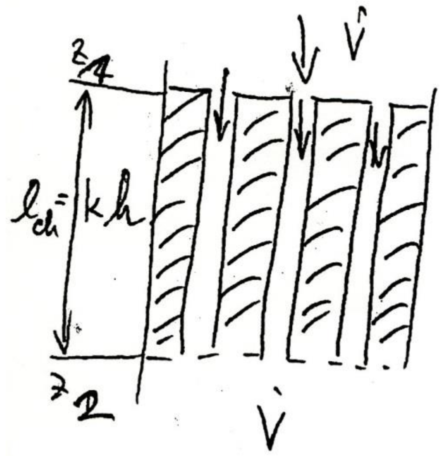
for non-spherical particles $a = (1-\varepsilon)\frac{6}{\psi d_p}$

ψ … sphericity, a measure of non-spherical shape ($\psi < 1$)

d_p … characteristic size (= diameter for sphere)

Flow in real bed is difficult to describe, therefore we use a model.

Channel model



We assume that void fraction and specific surface are the same for the channels and the real porous bed. Flow in channel can be described by Bernoulli equation.

$$\frac{v_{ch}^2}{2} + gz_1 + \frac{p_1}{\rho} = \frac{v_{ch}^2}{2} + gz_2 + \frac{p_2}{\rho} + e_{dis}$$

$$p_1 - p_2 = \Delta p = \underbrace{\rho g(z_2 - z_1)}_{\approx 0} + \rho e_{dis}$$

where $e_{dis} = \lambda \frac{l_{ch}}{d_{equiv}} \frac{{v_{ch}}^2}{2}$.

It can be shown that: $v_{ch} = \frac{hv}{\varepsilon}$, $d_{equiv} = \frac{4S_{ch}}{s_{ch}} = \frac{4\varepsilon}{a}$

On substitution:

$$e_{dis} = \lambda \frac{3}{2} \frac{kh(1-\varepsilon)}{\varepsilon \psi d_p} \frac{k^2 v^2}{2\varepsilon^2} = \frac{3}{2} \lambda k^3 \frac{(1-\varepsilon) v^2}{2\psi \varepsilon^3 d_p} h = \frac{3}{2} \lambda_S \frac{(1-\varepsilon) v^2}{2\psi \varepsilon^3 d_p} h$$

$$\Delta p = \rho e_{dis} = \frac{3}{2} \lambda_S \rho \frac{(1-\varepsilon) v^2}{2\psi \varepsilon^3 d_p} h$$

λ_S … friction factor in porous bed

$$\lambda_S = \underbrace{\frac{A}{\mathrm{Re}}}_{\substack{laminar \\ flow}} + \underbrace{B}_{\substack{turbulent \\ flow}} \quad \ldots Ergun\ equation, A = 133, B = 2.34$$