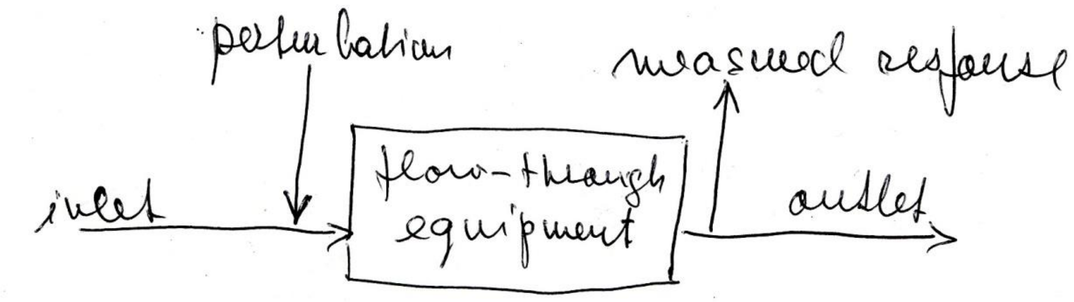
Character of fluid flow in equipment using residence time distributions

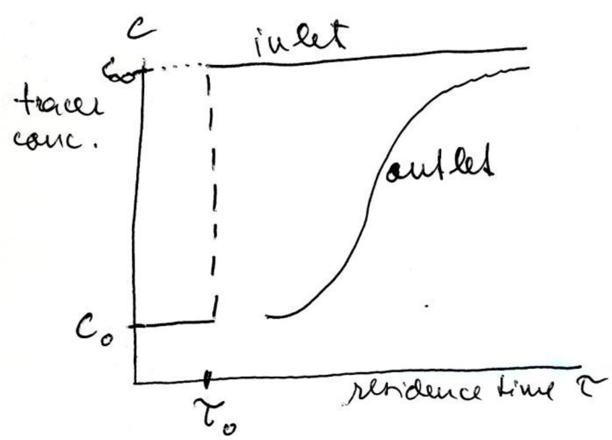
Due to turbulence, fluid flow through various equipment (pipes or stirred tanks with/without baffles, packed beds, cascade of stirred tanks) is usually random. “Particles” of the fluid (i.e. elements of continuum, see hydrodynamics) can have very different residence times in the equipment depending on the degree of mixing. In tubes and packed beds, the residence times tend to be similar. In stirred vessels, however, they span a vast range of values. In general, there is a distribution of residence times, which in practice can be assessed by employing the **method of perturbation and response**.

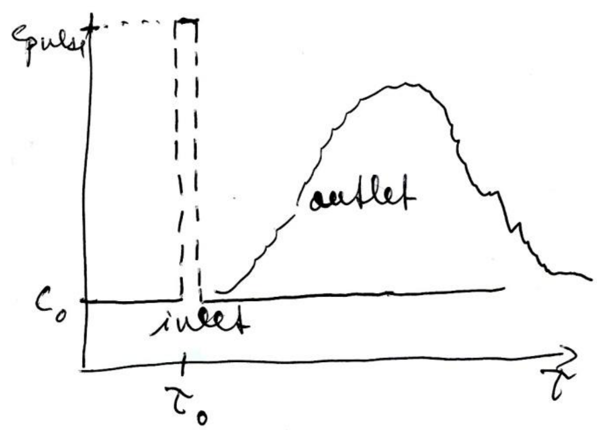


Typically, the perturbation is realized by introducing a **tracer** – a soluble component whose concentration is measured at the outlet. Particles (molecules) of the tracer are carried by fluid particles and represent their occurrence in the equipment and at the outlet.

There are **two basic types** of perturbations:

(i) **permanent switch** –introducing the tracer from certain time onwards
(ii) **pulse** – introducing the tracer for a short time only (limiting case of finite amount delivery within infinitesimal time is referred to as **impulse**)





τ_0 is normally chose to be 0

c_0 is typically 0 (but may be > 0)

c_∞ is gradually approached at the outlet (in theory) c_∞ is reached for $\tau \to \infty$)

c_{pulse} may be arbitrary; it determines the amount of tracer added within the pulse time $\Delta\tau$

Theoretical description

Residence time distribution (RTD) function $F(\tau)$ is defined as suggested by the permanent switch response

$$F(\tau) = \frac{c - c_0}{c_\infty - c_0}; \;\; 0 \leq F \leq 1$$

$F(\tau)$ can be interpreted as the probability that fluid particles will stay in the system within the time interval from 0 to τ. τ is the interpreted as the residence time. In other words, $F(\tau)$ determines a fraction of fluid particles having their residence times from 0 to τ, that is, particles, which left the system by the time τ. Within a differential time interval from τ to $\tau + \mathrm{d}\tau$ this fraction is $\mathrm{d}F$.

Differential RTD function $E(\tau)$ (also called density of RTD) is defined so that

$$\mathrm{d}F = E\,\mathrm{d}\tau \text{ or } E = \frac{\mathrm{d}F}{\mathrm{d}\tau}$$

Consequently

$$F(\tau) = \int_0^\tau E(\tau)\mathrm{d}\tau$$

$F(\tau)$ is non-decreasing function with maximum value $F(\tau \to \infty) = 1$ and thus

$$\int_0^1 \mathrm{d}F = \int_0^\infty E(\tau)\mathrm{d}\tau = 1$$

$E(\tau)$ is proportional to the measured outlet concentration:

$$E = k(c - c_0)$$

where

$$k = \frac{volumetric\ flow}{moles\ of\ injected\ tracer}$$

Mean residence time $\bar{\tau}$ is the integral mean value:

$$\bar{\tau} = \int_0^1 \tau \mathrm{d}F = \int_0^\infty \tau E(\tau)\mathrm{d}\tau$$

Alternatively, $\bar{\tau}$ can be readily calculated form the expression

$$\bar{\tau} = \frac{m}{\dot{m}} \text{ or for constant density } \bar{\tau} = \frac{V}{\dot{V}}$$

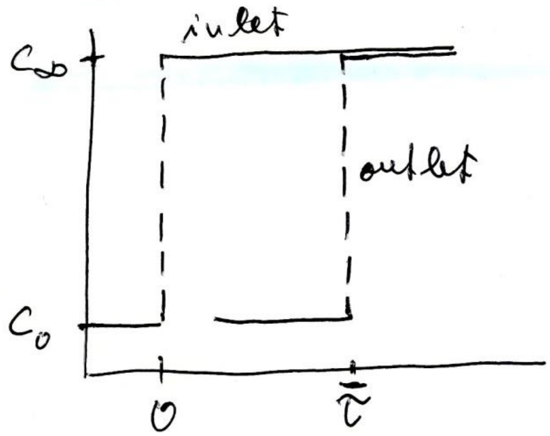
where m, V are mass and volume of the non-stagnant fluid in the equipment and $\dot{m}, \dot{V}$ are corresponding flows through the equipment.

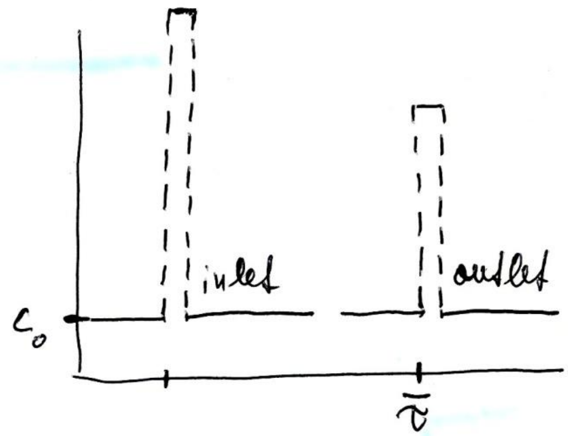
There are two limiting cases of flow patterns in equipment, plug flow and ideal mixing

Plug flow

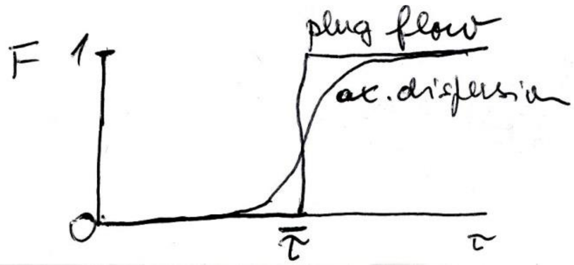
$$F(\tau) = \begin{cases} 0, & \tau < \bar{\tau} \\ 1, & \tau \geq \bar{\tau} \end{cases}$$

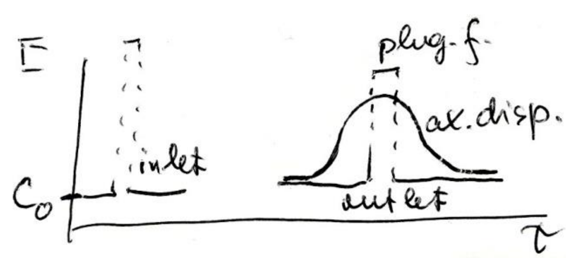
All particles have the same residence time, which is simultaneously mean residence time. The response signal has the same shape as the input signal but shifted by $\bar{\tau}$.





Plug flow is a basic flow pattern assumed in the flow through tubes and packed beds. In reality, it can be disturbed by **axial dispersion** – $F(\tau)$ then becomes flatter and $E(\tau)$ broadens.





Ideal mixing

Concentration, temperature and other intensive properties are the same everywhere in the equipment but residence times of fluid particles vary smoothly. The function $F(\tau)$ can be derived from the unsteady mass balance of an ideal mixer subjected to permanent switch tracer experiment:

inlet = outlet + accumulation

$$\dot{V}c_\infty = \dot{V}c + V\frac{\mathrm{d}c}{\mathrm{d}\tau}$$

initial condition: at $\tau = 0 \quad c = c_0$

This corresponds to transient evolution of the system from the instant of the switch onwards. The equation is solved by separation + integration.

$$\int_{c_0}^{c} \frac{\mathrm{d}c}{c_\infty - c} = \frac{\dot{V}}{V}\int_0^\tau \mathrm{d}\tau = \frac{1}{\bar{\tau}}\int_0^\tau \mathrm{d}\tau$$

$$\ln\frac{c_\infty - c}{c_\infty - c_0} = -\frac{\tau}{\bar{\tau}} \rightarrow \frac{c_\infty - c}{c_\infty - c_0} = \exp\left(-\frac{\tau}{\bar{\tau}}\right)$$

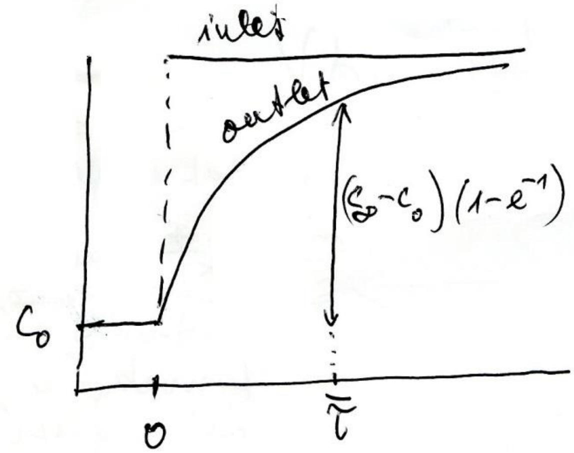
The RTD function $F(\tau)$ is now calculated as

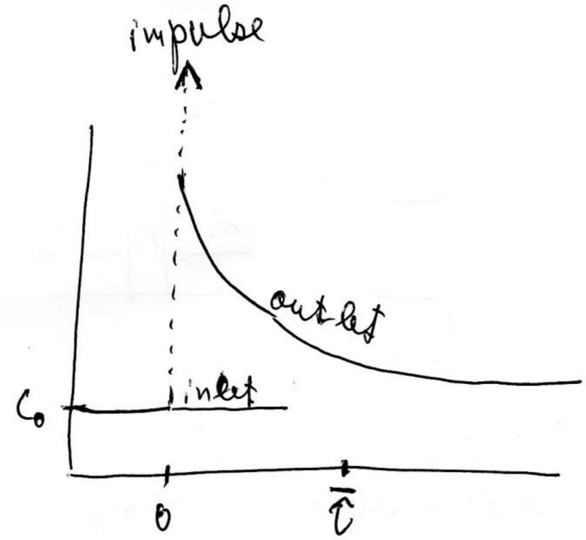
$$F(\tau) = 1 - \frac{c_\infty - c}{c_\infty - c_0} = 1 - \exp\left(-\frac{\tau}{\bar{\tau}}\right)$$

Then the differential RTD function $E(\tau)$ is

$$E(\tau) = \frac{\mathrm{d}F(\tau)}{\mathrm{d}\tau} = \frac{1}{\bar{\tau}}\exp\left(-\frac{\tau}{\bar{\tau}}\right)$$

Both response curves are thus exponentials.





General flow pattern is somewhere between the two limiting cases. For example, RTD in a cascade of ideal mixers approaches the plug flow pattern as the number of mixer grows to infinity.