Sedimentation (settling)

- separation of particles disperse in a medium by an external force

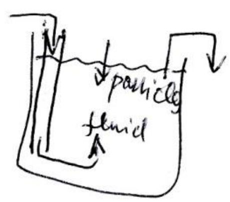
External forces: mostly gravitational, can be also centrifugal, electric or magnetic

Types of dispersions

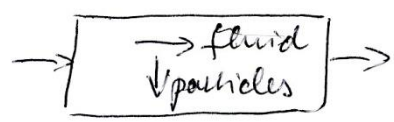
- suspension: solids in liquid medium
  solids in gaseous medium
- mist: liquid in gaseous medium
- emulsion: liquid in liquid medium

Equipment

- gravitational settlers: vertical flow



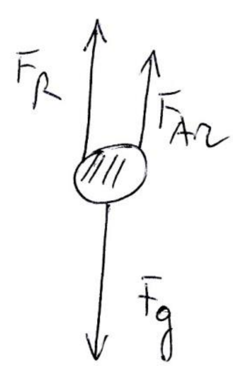
horizontal flow



- rotational settlers: centrifuge
  cyclone
- electrostatic or magnetic separators

Gravitational settling

forces action a particles

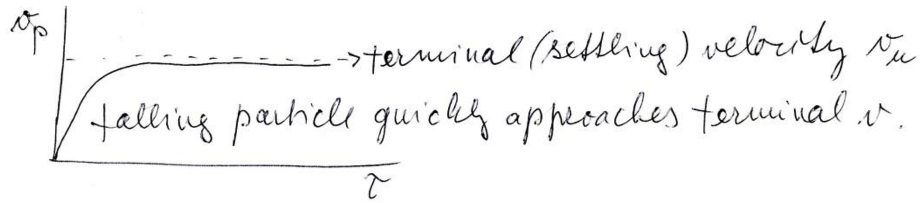


- **external**: F_g … **gravitational**
- **internal**: $F_A = \underbrace{F_{Ar}}_{\substack{\boldsymbol{buoyant} \\ (Archimedes)}} + \underbrace{F_R}_{\substack{\boldsymbol{drag} \\ (resistance)}}$

Newton second law (= momentum balance)

$$-F_g + F_{Ar} + F_R = m_p \frac{\mathrm{d}v_p}{\mathrm{d}\tau}$$

v_p … velocity of the particle



At steady state $\frac{\mathrm{d}v_p}{\mathrm{d}\tau} = 0 \;\rightarrow\; v_p = v_u$ ... terminal (settling velocity)

- balance of forces

$$F_g - F_{Ar} = F_R$$

$$\rho_p g V_p - \rho_f g V_p = \zeta_u \rho_f \frac{v_u^2}{2} S_p$$

for a sphere $V_p = \pi \frac{d_p^3}{6}$, $S_p = \pi \frac{d_p^2}{4}$ projected area in the direction of the fall

$$(\rho_p - \rho_f) g \pi \frac{d_p^3}{6} = \zeta_u \rho_f \frac{v_u^2}{2} \pi \frac{d_p^2}{4}$$

thus

$$\boldsymbol{v_u^2 = \frac{4}{3} \frac{g d_p}{\zeta_u} \frac{\rho_p - \rho_f}{\rho_f}} \qquad \textbf{(Eq. A)}$$

ζ_u … drag coefficient

ρ_p, ρ_f … densities of particles and fluid

It is convenient to rewrite Eq. A in a dimensionless form upon introducing:

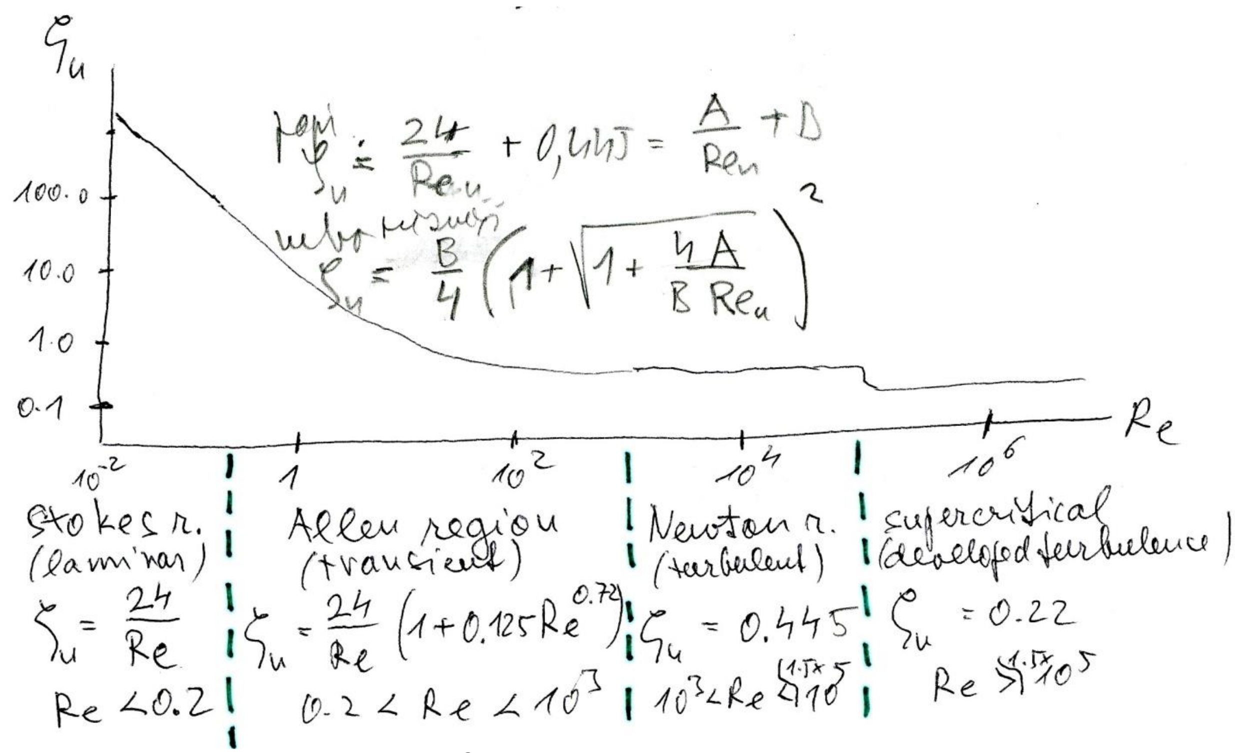
- **Reynolds number**: $\mathrm{Re} = \frac{v_u d_p \rho_p}{\eta} = \frac{v_u d_p}{\nu}$
- **Archimedes number**: $\mathrm{Ar} = \frac{g d_p^3 (\rho_p - \rho_f) \rho_f}{\eta^2} = \frac{g d_p^3 (\rho_p - \rho_f)}{\nu^2 \rho_f}$

On multiplying Eq. A by $\frac{d_p^2}{\nu^2}$ we obtain

$$\frac{d_p^2}{\nu^2} v_u^2 = \frac{4}{3\zeta_u} \frac{g d_p^3}{\nu^2} \frac{\rho_p - \rho_f}{\rho_f}$$

$$\mathrm{Re}^2 = \frac{4}{3\zeta_u}\, \mathrm{Ar} \rightarrow \mathbf{Ar} = \boldsymbol{\frac{3}{4} \zeta_u} \mathbf{Re^2}$$

However, the drag coefficient ζ_u depends on Re (similarly as the friction factor λ depends on Re in fluid flow in pipes) and hence this relation must be given → obtained empirically (graph or ”correlations” between ζ_u and Re) .



The dimensionless force balance $\mathrm{Ar} = \frac{3}{4}\zeta_u \mathrm{Re}^2$ in particular region reads:

a) Stokes region:

$$\mathbf{Ar = 18Re};\ \mathrm{Re}<0.2,\ \mathrm{Ar}<3.6$$

b) Allen region

$$\mathbf{Ar = 18Re(1+0.125Re^{0.72})}$$

$$0.2 < \mathrm{Re} < 10^3;\ 3.6 < \mathrm{Ar} < 3.43\times10^5$$

c) Newton region

$$\mathbf{Ar = 0.33Re^2}$$

$$10^3 < \mathrm{Re} < 1.5 \times 10^5;\ 3.43\times10^5 < \mathrm{Ar} < 7.4\times10^9$$

Discussion:

There are two tasks:

- d_p is given and we want to find v_u
  If d_p is given then Re can be expressed from a) or c) and. Re cannot be expressed from b) and its value must be obtained by **iteration**

- v_u is given and we want to find d_p
  Iteration is required in all three cases (that is, d_p is estimated → Re → Ar → new estimate ...)

We can avoid iteration by introducing **Lyaschenko number**:

$$Ly = \frac{v_u^3 \rho_f^2}{g\eta(\rho_f - \rho_p)} = \frac{v_u^3 \rho_f}{g\nu(\rho_f - \rho_p)}$$

This number contains only vu and not dp (unlike Re), then:

a) Stokes: $\mathrm{Ar} = 76.4\ \mathrm{Ly}^{\frac{1}{2}}$
c) Newton: $\mathrm{Ar} = 0.036\ \mathrm{Ly}^{\frac{1}{2}}$

In Allen region we can use a less accurate correlation

b) Allen: : $\mathrm{Ar} = 138.6\ \mathrm{Ly}^{0.875}$

Remark: Terminal velocity of a nonspherical particle can be found by using:

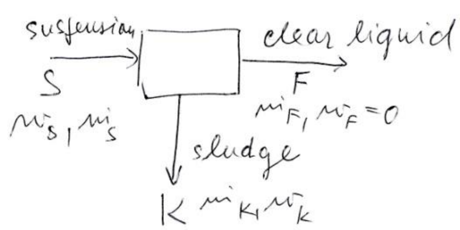
1) **dynamical shape factor** φ_{Ar} according to $v_u = \varphi_{Ar} v_{u,0}$
   $v_{u,0}$ … terminal velocity of an equivalent spherical particles having the same volume as nonspherical particle
   φ_{Ar} is an empirical function of a factor $F = {l_p}/{l_d}$ and Ar

   l_p … maximum length of nonspherical particle
   l_p … diameter of equivalent spherical particle
   Ar is evaluated using dp
2) **sphericity** $\psi = {A_0}/{A}$
   A_0 … surface area of equivalent spherical particle
   A … surface area of nonspherical particle
   v_u is found from Ly which is given graphically as function of Ar and

Design of a settler

Calculation of settling area

First step is **mass balance** (notation S, F, K as in filtration)



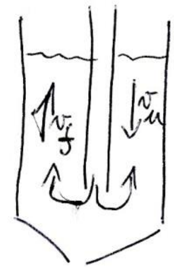
- total mass balance
$$\dot{m}_S = \dot{m}_F + \dot{m}_K$$
- balance on solids
$$\dot{m}_S w_S = \dot{m}_K w_K$$
$$\dot{m}_F = \dot{m}_S (1 - \frac{w_S}{w_K})$$

Then

$$\dot{V}_F = \dot{m}_F \rho_f$$

Remark: Balance on volume can be also used

Settling area:

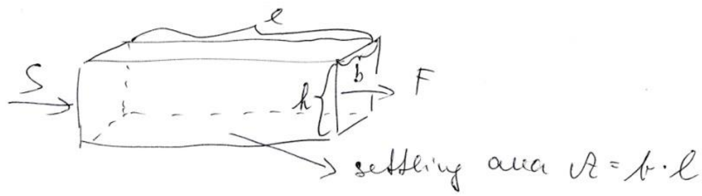


- **settler with vertical flow**

  condition: the fluid average velocity $v_f = \frac{\dot{V}_F}{A}$ must be equal to the terminal velocity

  $v_f = \frac{\dot{V}_F}{A} = v_u \rightarrow A = \frac{\dot{V}_F}{v_u}$, where $A = \frac{\pi D^2}{4}$ is the area for settling

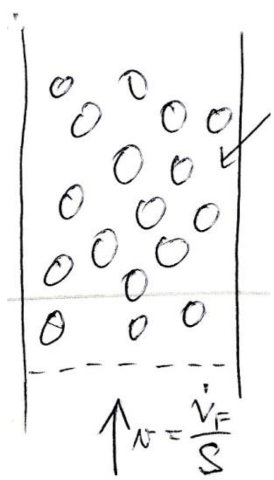
- **settler with horizontal flow**

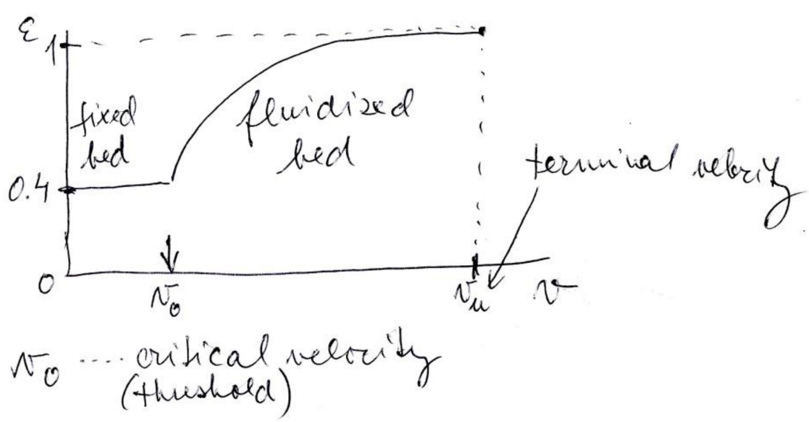


  condition: settling time $\tau_u = \frac{h}{v_u}$ must be equal to the mean residence time for the fluid $\bar{\tau} = \frac{V}{\dot{V}} \doteq \frac{hlb}{\dot{V}_F}$, thus $A = \frac{\dot{V}_F}{v_u}$

Fluidization

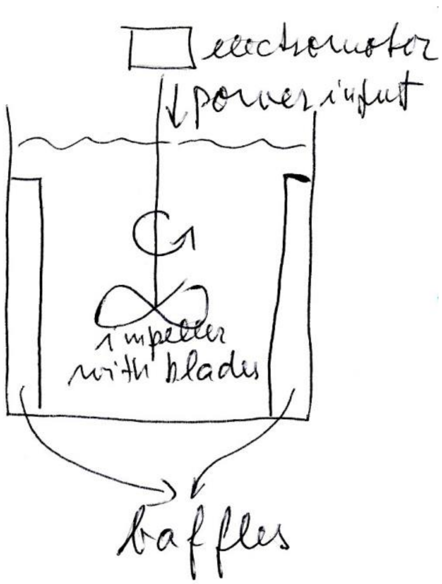
- bed of solid particles kept in fluidized state by flow of a fluid from below – useful in rapid mass/ heat exchange and in heterogeneous reactors
- void fraction depends on the superficial velocity v





Mixing

- homogenization of mixtures
- typically mixing is achieved by using a stirrer (= impeller)
- to enhance mixing, baffles are used



Calculation of power input

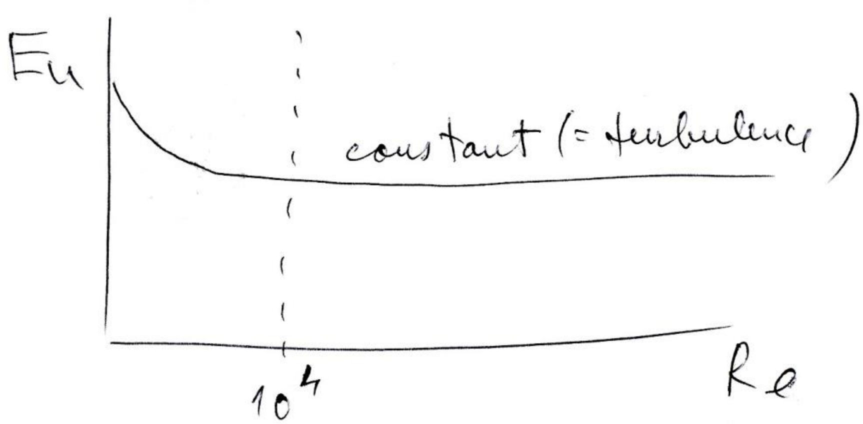
Euler number: $\mathrm{Eu} = \frac{P}{n^3 d^5 \rho}$

Reynolds number: $\mathrm{Re} = \frac{n d^2 \rho}{\eta}$

Geometrical simplex: Γ, e.g number of blades

n … frequency of stirring

d … diameter of the impeller



Dimensional analysis

Dimensional equation are transformed to dimensionless equations → new dimensionless parameters in these equation are for example:

- Reynolds number
- Archimedes number
- Friction factor
- Drag coefficient

and many more.

Advantage is, that:

- description in terms of dimensionless quantities is more universal
- solutions of the equations, which may be hard to obtain are replaced by empirical relations such as:

$$\lambda = \lambda(\mathrm{Re}, {}^{\varepsilon}/_{d})$$
$$\zeta_u = \zeta_u(\mathrm{Re})$$
$$\mathrm{Ar} = \mathrm{Ar}(\mathrm{Ly})$$
$$\mathrm{Eu} = \mathrm{Eu}(\mathrm{Re}, \Gamma_1, \Gamma_2, \dots)$$