Drying

- evaporation of a liquid (moisture) from a mixture with a solid

In drying mass and heat transfer occur simultaneously. The solid is a porous granular material soaked with liquid, which may cover external surface of the solid particles (provided there are any).

Equipment for drying → **dryer**

a) **direct (adiabatic) dryers** – the solid is exposed to a hot gas (usually air) which is preheated in a preheater (furnace, in many non-English languages the term kalorifer or calorifer is used for the preheater)
b) **indirect (nonadiabatic) dryers** – the solid is heated through the metal wall of the dryer by external medium (steam, also by radiation, for example by microwaves)

Dryers can be operated:

a) **batchwise** – tray dryers, fluid-bed dryers
b) **continually** – screen-conveyor dryers, rotary turbine dryers, rotary drum dryers, also fluid-bed dryers

Notation for components

A … humidity (in the gas) or moisture (in the solid)

B … dry gas

C … dry solid

We assume that only A is transferred from the (moist) solid to the (humid) gas. Therefore composition in both phases is conveniently described in terms of **relative mass fractions:**

- in gas: $U_A = \frac{m_A}{m_B}$ … normally called humidity
- in solid: $W_A = \frac{m_A}{m_C}$ … normally moisture content

In gas, instead of humidity one can also express the composition by **relative humidity**:

$$\varphi_A = \frac{p_A}{p_A^0}$$

p_A is **partial pressure** of A

p_A^0 is **vapor pressure** (i.e. equilibrium or saturated conditions in the gas)

$p_A \leq p_A^0$ and thus φ_A ranges from 0 to 1 (or from 0 % to 100 %)

Remark: p_A^0 can be found in steam stables or via Antoine equation.

In equilibrium between wet solid and wet gas the moisture in the solid can be:

a) **bound** – i.e. A interacts with the molecules of the solid (in the pores or close to the outer surface)
b) **unbound** – i.e. A forms a continuous layer covering C → very wet solid

Equilibrium curve: $U_A = f(W_A)$ or $\varphi_A = f(W_A)$

$\varphi_A = 1$ bound   unbound $\varphi_A = f\left(W_A\right)$

W_{AC} ··· critical moisture content W_A
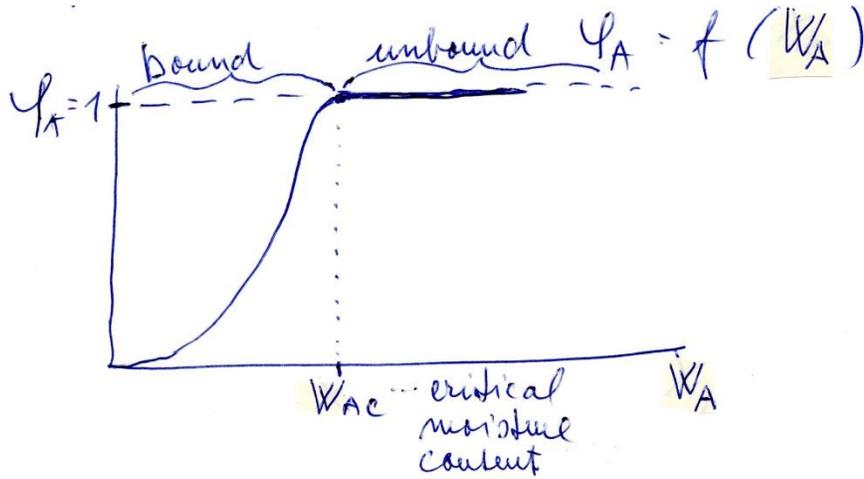
Humidity chart of a humid gas

It is a plot of relative specific enthalpy I versus humidity U_A complemented by lines (curves) of constant other properties of the gas

$$I = \frac{H_g}{m_B}$$

where H_g is (normal) enthalpy of gas:

$$H_g = m_A h_A + m_B h_B \;\rightarrow\; I = U_A h_A + h_B$$

choosing a reference state: A is liquid and B is gas at $t_0 = 0\,°C$ we write:

$$h_A = \Delta h_{vap,A0} + c_{pA,g} t$$

$$h_B = c_{pB} t$$

so finally

$$I = U_A \Delta h_{vap,A0} + \underbrace{\left(U_A c_{pA,g} + c_{pB}\right)}_{C_p} t$$

where

$\Delta h_{vap,A0}$ is specific latent heat of evaporation [J/kg]

$c_{pA,g}$ is mean specific heat capacity of gaseous A at $\langle t \rangle = \frac{t+t_0}{2} = \frac{t}{2}$ [J/kg/k]

c_{pB} is mean specific heat capacity of B at $\langle t \rangle = \frac{t}{2}$ [J/kg/K]

C_p is called **relative specific heat capacity**

Analogously in the wet solid we have:

$$I_m = \underbrace{\left(W_A c_{pA,l} + c_{pC}\right)}_{C_{pm}} t + \underbrace{\frac{\Delta H_{mix}}{m_C}}_{\doteq 0} \;\; \text{we assume ideal mixture}$$
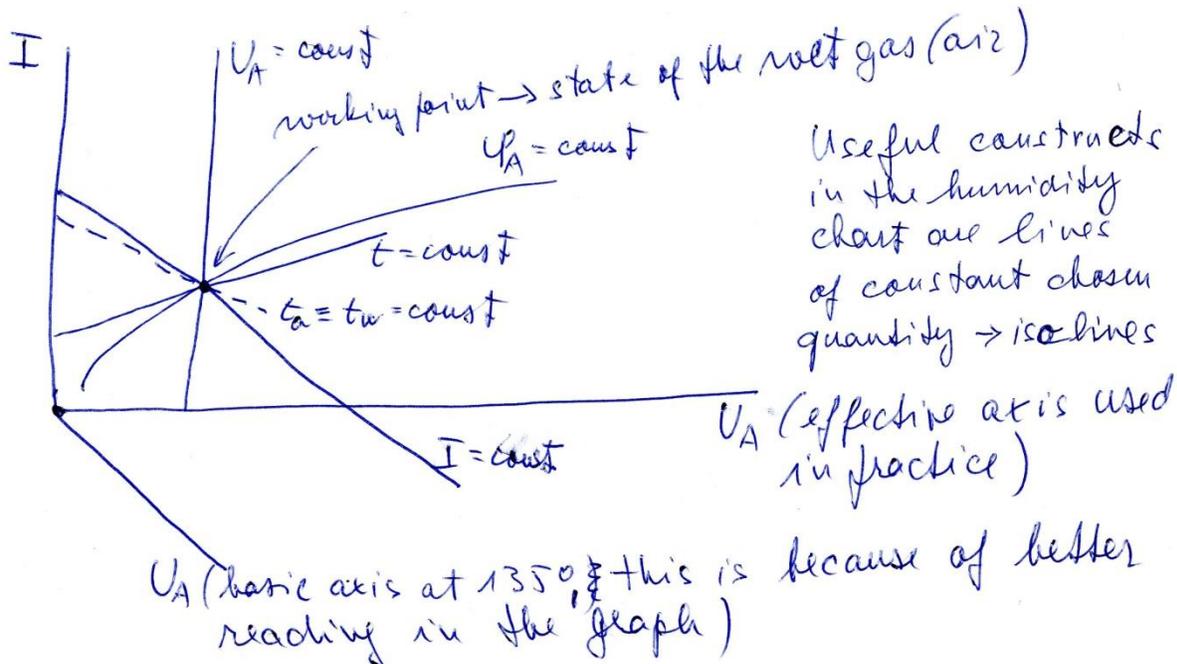
where

$c_{pA,l}$ is mean specific heat capacity of liquid A at $\langle t \rangle = \frac{t}{2}$ [J/kg/k]

c_{pC} is mean specific heat capacity of C at $\langle t \rangle = \frac{t}{2}$ [J/kg/K]

C_{pm} is relative specific heat capacity of wet solid (material)

Sketch of the humidity chart

Useful constructs in the humidity chart are lines of constant choses quantity → isolines

Here: $t_a = t_W$ → temperature of adiabatic saturation = temperature of the wet-bulb thermometer

Any point (**working point**) in the graph can be found as **intersection** of an arbitrary **pair of chosen lines/curves.**

Remark: *working point at* $\varphi_A = 100\ \%$ *is **dew point***

Wet bulb thermometer serves as a measurement device for determining t_W to be used in finding the working point (the other measure might typically be (normal) temperature t). Wet bulb thermometer is a thermometer with bulb covered by a wet cloth. When immersed in flow of humid gas, its temperature drops to a lower value t_W than the temperature of the gas t. This is because of release of latent heat from the cloth to the air, which is compensated by heat flow from the gas. Simultaneously mass transfer accompanies the evaporation.
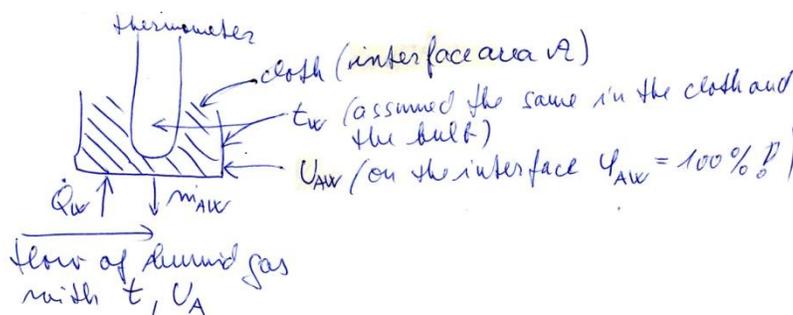
At steady state:

- **enthalpy balance**: $\quad \dot{Q}_W = \dot{m}_{AW} \Delta h_{vap,A,W}$
- **mass transfer equation**: $\quad \dot{m}_{AW} = K_U(U_{AW} - U_A)A$
- **heat transfer equation**: $\quad \dot{Q}_W = \alpha(t - t_W)A$

By combining the balance and transfer equations:

$$t - t_W = \frac{K_U \Delta h_{vap,A,W}}{\alpha}(U_{AW} - U_A)$$

That is, there is a linear relation between driving forces for heat and mass transfer

Conclusion

Although the **wet-bulb temperature** t_W is a **dynamical** (=nonequilibrium) property, it can be shown that for humid air it is equivalent to a **thermodynamic** (=equilibrium) property called **temperature of adiabatic saturation** t_a, which in turn is used in the humidity chart. So we can use t_W (which is easily measured) instead of t_a (inconvenient for measurements).

Remark: A device combining (normal) thermometer and a wet-bulb thermometer is called **psychrometer**.

Continuous counter-current dryer with preheater
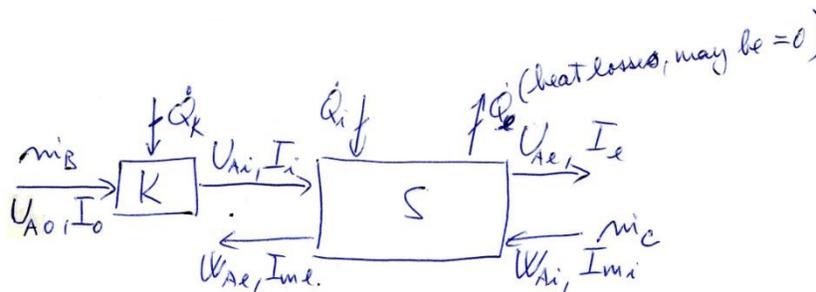
K … preheater

S … dryer

$\dot{Q}_K > 0$ and $\dot{Q}_i = 0$ … direct (adiabatic) dryer (if)

$\dot{Q}_K = 0$ and $\dot{Q}_i > 0$ … indirect (nonadiabatic) dryer

$\dot{m}_B$ … flow of dry gas is constant throughout

$\dot{m}_C$ … flow of dry solid is also constant

Mass balance in K: $\quad \dot{m}_B U_{A0} = \dot{m}_B U_{Ai} \rightarrow U_{A0} = U_{Ai}$

Enthalpy balance in K: $\dot{m}_B I_0 + \dot{Q}_K = \dot{m}_B I_i \rightarrow I_i - I_0 = \frac{\dot{Q}_K}{\dot{m}_B}$

Mass balance in S: $\quad \dot{m}_B(U_{Ae} - U_{Ai}) = \dot{m}_C(W_{Ai} - W_{Ae}) = \dot{m}_A$ … mass flow of evaporated liquid

Enthalpy balance in S: $\dot{m}_B(I_e - I_0) = \dot{m}_C(I_{mi} - I_{me}) + \dot{Q}_i - \dot{Q}_e$
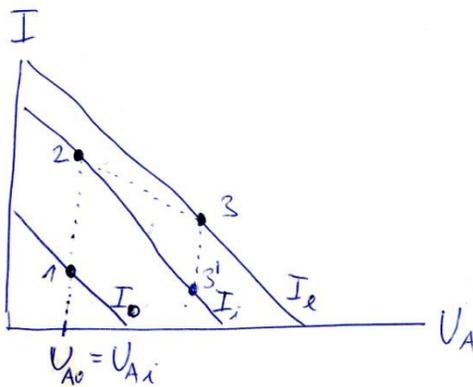
Combined enthalpy balance in K+S:

$$(I_e - I_0) = \frac{\dot{m}_C}{\dot{m}_B}(I_{mi} - I_{me}) + \frac{q}{l}$$

where

$q = \frac{\dot{Q}_K + \dot{Q}_i - \dot{Q}_e}{\dot{m}_A}$ ... specific heat (heat duty) per mass of evaporated liquid
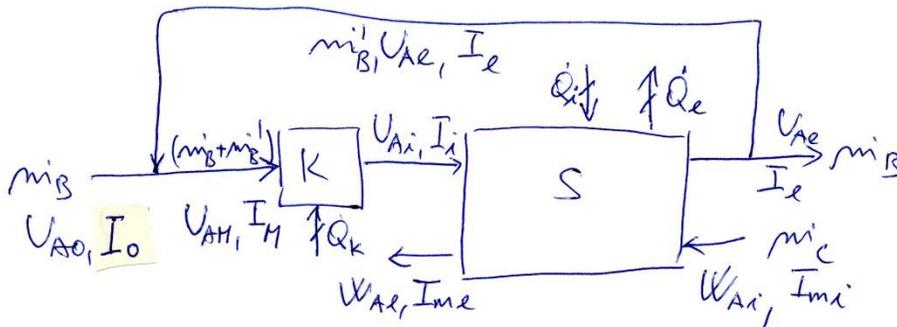
$l = \frac{\dot{m}_B}{\dot{m}_A}$ ... specific consumption of drying gas

Graphical representation of gas during the drying operation:

1. working point corresponding to input to K
2. working point corresponding to input to S
3. working point corresponding to output from S
4. working point corresponding to output from S
   for an **ideal dryer**, which is defined by $I_i = I_e$

Dryer with recycling gas

We define the recycle parameter: $R = \frac{\dot{m}'_B}{\dot{m}_B}$

Mass and enthalpy balances of the node where fresh gas mixes with the recycling gas imply **lever rule** in the humidity chart.
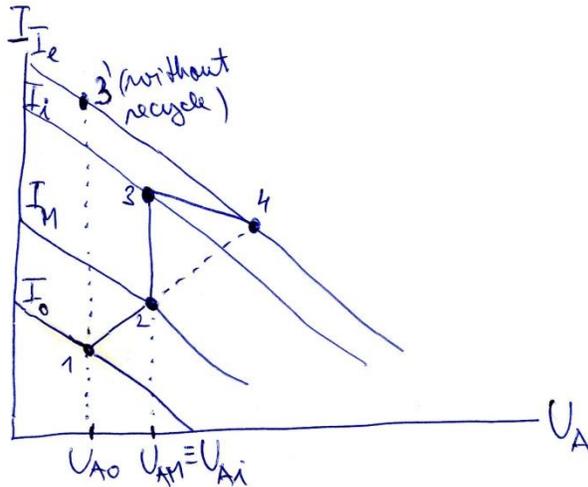
mass balance of A:

$$\dot{m}_B U_{A0} + \dot{m}'_B U_{Ae} = (\dot{m}_B + \dot{m}'_B)U_{AM}$$

$$U_{A0} + R U_{Ae} = (1 + R)U_{AM}$$
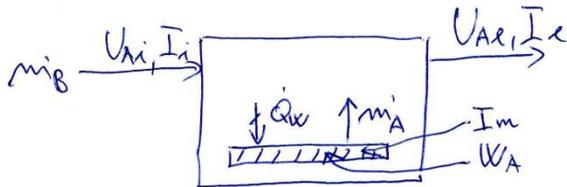
enthalpy balance

$$I_0 + RI_e = (1 + R)I_M$$

By lever rule points 1, 2 and 4 must lie on a straight line and $R = \dfrac{\dot{m}'_B}{\dot{m}_B} = \dfrac{\overline{12}}{\overline{24}}$

1) input of fresh gas
2) fresh gas mixed with recycle
3) preheated gas input to S
4) preheated gas without recycle → higher temperatures → may damage thermolabile material
5) gas in the recycle and at the output from S

Batch drying

Moist solid is placed on a tray/shelves and exposed to drying for certain time that we used to determine. Notice that the system is batch for the moist solid but continual for the gas.

$\dot{m}_A$ ... mass flow of liquid to the gas

$\dot{Q}_W$ ... heat flow to the material

A ... surface of dried material

m_C ... mass of dry solid

m_A ... mass of liquid in the moist solid

$$m_A = m_C W_A$$

Balances of the moist solid

- mass of A: $\quad 0 = \dot{m}_A + \dfrac{dm_A}{d\tau} = \dot{m}_A + m_C \dfrac{dW_A}{d\tau}$

- enthalpy: $\quad \dot{Q}_W = \dot{m}_A h_{vap,A} + m_C \dfrac{dI_m}{d\tau}$
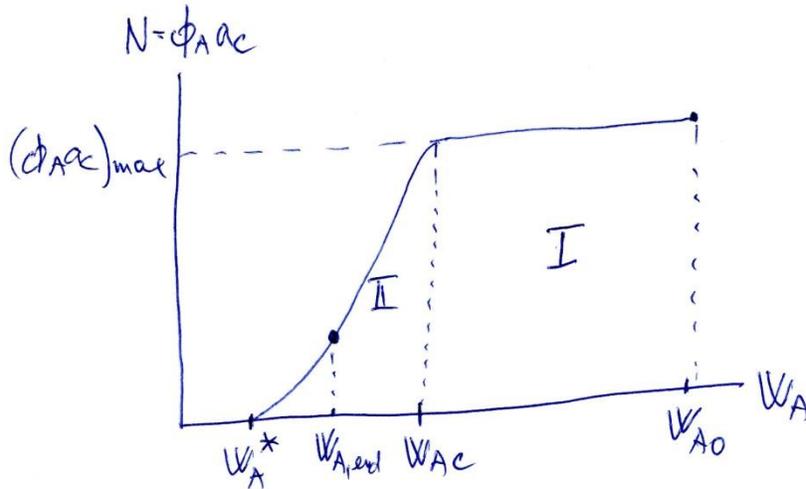
Rewrite mass balance as follows:

$$N = \underbrace{\frac{\dot{m}_A}{m_C}}_{\substack{drying \\ rate}} = \underbrace{\frac{\dot{m}_A}{A}}_{\substack{drying \\ flux}} \underbrace{\frac{A}{m_C}}_{\substack{specific \\ surface}} = \Phi_A a_C = -\frac{dW_A}{d\tau}$$

Remarks:

- The drying rate $N = \Phi_A a_C$ is measured as function of W_A for each material, $N = N(W_A)$
- Mass transfer implies: $\dot{m}_A = k_U(U_{Aw} - U_A)A$

The plot of $N = \Phi_A a_C$ versus W_A specifies the kinetics of drying:

I ... period of constant drying rate

II ... period of falling drying rate

Period I: above critical moisture content; evaporation from free surface of liquid covering the solid, situation equivalent to that of the wet-bulb thermometer → temperature of the moist solid = t_W, which can be found for the gas from the humidity chart.

Period II: below critical moisture content; the moisture must be transported from the pores to the surface which slows down the drying rate; temperature of the moist solid $< t_W$.

The **equilibrium moisture content** W_A^* is nonzero for humid gas and cannot be reached in a finite time → drying is stopped at a moisture content $W_{A,end} > W_A^*$

Calculation of the drying time

- by integration:

$$\tau = -\int_{W_{A,0}}^{W_{A,end}} \frac{dW_A}{N} = \int_{W_{A,end}}^{W_{A,0}} \frac{dW_A}{N}$$

where $N = N(W_A)$. The calculation is split into two parts:

1) period of constant rate $\left(W_{A,c} \leq W_A \leq W_{A,0}\right)$:

$$\tau_I = \int_{W_{A,c}}^{W_{A,0}} \frac{dW_A}{(\Phi_A a_C)_{max}} = \frac{W_{A,0} - W_{A,c}}{(\Phi_A a_C)_{max}} = \frac{W_{A,0} - W_{A,c}}{k_U(U_{Aw} - U_A)a_c}$$

2) period of falling rate $\left(W_{A,end} \leq W_A \leq W_{A,c}\right)$

- here $N = N(W_A)$ may be given by table (numerical integration) or we can assume **linear drop** of the drying rate

$$N = (\Phi_A a_C)_{max} \frac{W_A - W_A^*}{W_{A,c} - W_A^*}$$

then

$$\tau_{II} = \int_{W_{A,c}}^{W_{A,0}} \frac{dW_A}{(\Phi_A a_C)_{max} \dfrac{W_A - W_A^*}{W_{A,c} - W_A^*}} = \frac{W_{A,c} - W_A^*}{(\Phi_A a_C)_{max}} \ln \frac{W_{A,c} - W_A^*}{W_{A,end} - W_A^*}$$

Finally:

$$\tau = \tau_I + \tau_{II}$$