

# Heat processes

## Heat exchange

**Heat** – energy transported across a surface from higher temperature side to lower temperature side; it is a macroscopic measure of transported energies of molecular motions

**Temperature** – a macroscopic measure of kinetic energies of molecules

## Mechanisms of heat transport

- i) **conduction** – transport mediated by individual molecules
- ii) **convection** – in fluids the transport of heat may occur so that large assemblies of molecules transport their energy as a whole
  - a) **free** convection – due to density difference
  - b) **forced** convection – due to pumping
- iii) **radiation** – mediated by electromagnetic waves

Next we express heat flow in two basic situations:

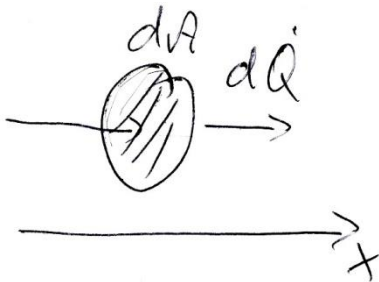
- heat flow in medium at rest
- heat flow in a flowing medium near interface
- heat flow radiated from hot body

## Heat flow in non-moving environment

- conduction of heat applies here

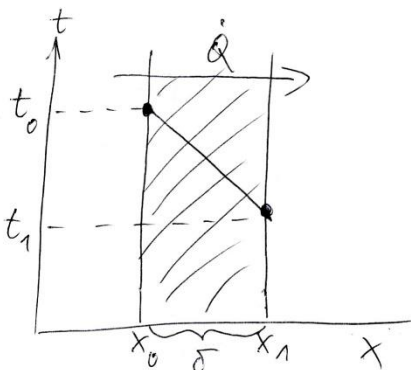
**Fourier law** (empirical observation)

$$d\dot{Q} = -\lambda \frac{dt}{dx} dA \text{ or } q = \frac{d\dot{Q}}{dA} = -\lambda \frac{dt}{dx}$$



- $\lambda$  ... thermal conductivity
- $\frac{dt}{dx}$  ... temperature gradient (= driving force)
- $q$  ... heat flux
- $\lambda \approx 10^{-2}$  ... thermal insulators
- $\lambda \approx 10^2$  ... thermal conductors (e.g. metals)

## Conduction in a plate:



$$\dot{Q} = -\lambda \frac{dt}{dx} A$$

at steady state

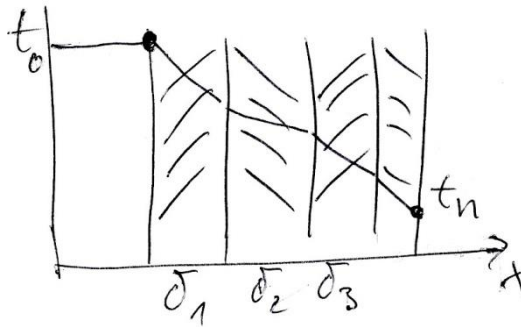
$$\dot{Q} = \text{const} \rightarrow \frac{dt}{dx} = \frac{\Delta t}{\Delta x} = \frac{t_1 - t_0}{x_1 - x_0} = \frac{t_1 - t_0}{\delta}$$

thus  $\dot{Q} = -\lambda \frac{t_1 - t_0}{\delta} A$

or

$$\dot{Q} = \frac{\overbrace{t_0 - t_1}^{\text{driving force}}}{\underbrace{\frac{\delta}{\lambda}}_{\text{resistance}}} A$$

multiple layers:



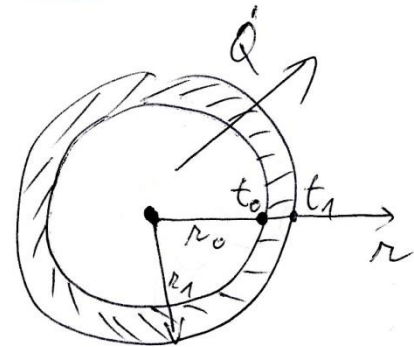
$$\dot{Q} = \frac{t_1 - t_n}{\sum_{i=1}^n R_i} A \quad \text{where } R_i = \frac{\delta_i}{\lambda_i} \dots \text{resistance in the } i\text{-th layer}$$

### Conduction in a cylinder

at steady state  $\dot{Q} = \text{const}$

$$\dot{Q} = -\lambda \frac{dt}{dr} A = -\lambda \frac{dt}{dr} 2\pi r L$$

$L$  ... length of the tube



by integration

$$\int_{r_0}^{r_1} \frac{dr}{r} = \int_{t_0}^{t_1} -\frac{2\pi L \lambda}{\dot{Q}} dt$$

$$\ln \frac{r_1}{r_0} = -\lambda \frac{2\pi L}{\dot{Q}} (t_1 - t_0); \left( \frac{r_1}{r_0} = \frac{d_1}{d_0} \right)$$

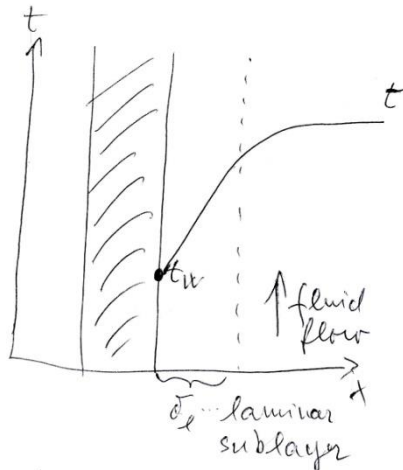
$$\text{or } \dot{Q} = 2\pi L \lambda \frac{(t_0 - t_1)}{\ln \frac{d_1}{d_0}} = \pi \frac{\overbrace{(t_0 - t_1)}^{\text{driving force}}}{\underbrace{\frac{1}{2\lambda \ln \frac{d_1}{d_0}}}_{\text{resistance}}} L$$

multiple layers

$$\dot{Q} = \pi \frac{(t_0 - t_1)}{\sum \frac{1}{2\lambda_i \ln \frac{d_i}{d_{i-1}}}} L$$

### Heat transfer from a flowing fluid to an interface with a solid wall

Heat transfer → Newton law of cooling (empirical)



$$d\dot{Q} = \alpha(t - t_w)dA \quad \text{or} \quad q = \frac{d\dot{Q}}{dA} = \alpha(t - t_w)$$

$\alpha \left[ \frac{W}{m^2K} \right]$  ... **heat transfer coefficient**

$(t - t_w)$  ... driving force

$t$  ... average temperature in the fluid

$t_w$  ... temperature at the wall

**Theoretically:**

$$q = \frac{\delta}{\lambda_l}(t - t_w)$$

but the thickness of the laminar sublayer  $\lambda_l$  is difficult to evaluate

**Instead:**  $\alpha$  is calculated by using dimensionless numbers and dimensionless relation (=correlation) obtained from experiments.

Remark:  $d\dot{Q} = \frac{(t-t_w)}{\frac{1}{\alpha}} \rightarrow \frac{1}{\alpha}$  is **resistance against heat transfer**

### Dimensionless numbers used

$Nu = \frac{\alpha l}{\lambda}$  ... Nusselt number ( $l$  is characteristic length)

$Pe = \frac{vl}{a}$  ... Péclet number ( $a = \frac{\lambda}{\rho c_p}$  ... thermal diffusivity)

$Re = \frac{vl}{\nu} = \frac{vl\rho}{\eta}$  ... Reynolds number

$Ar = \frac{gl^3\Delta\rho}{\nu^2\rho}$  ... Archimedes number

$Gr = \frac{gl^3}{\nu^2} \beta \Delta t$  ... Grashof number

$Pr = \frac{\nu}{a}$  ... Prandtl number (material properties)

$\alpha$  is calculated from Nu which in turn is calculated from a dimensionless correlation  
 $Nu = Nu(Re, Pe, Ar, Gr, Pr, \dots)$

**Examples of correlations:**

a) **free convection** (e.g. in pipes with large diameter)

$$Nu = C(GrPr)^n; \quad C, n \text{ are empirical values}$$

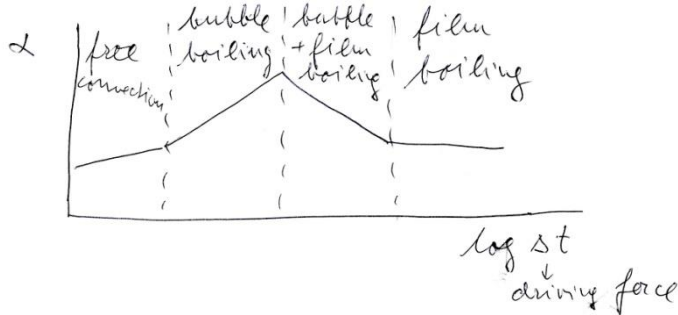
b) **forced convection**

$$Nu = 0.023Re^{0.8}Pr^{0.4}; \quad Re > 10000$$

c) **heat transfer in boiling liquids or condensing vapors**

correlations are complex but qualitative explanation is as follows

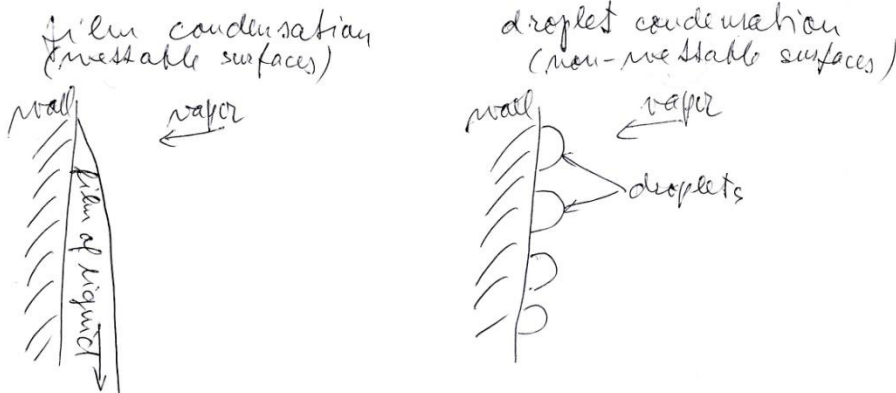
(i) **boiling**



**bubble boiling** = bubbles arise at the wall and rise

**film boiling** = bubbles at the wall merge into a film of vapor which has a low conductivity, therefore  $\alpha$  drops

(ii) **condensation**

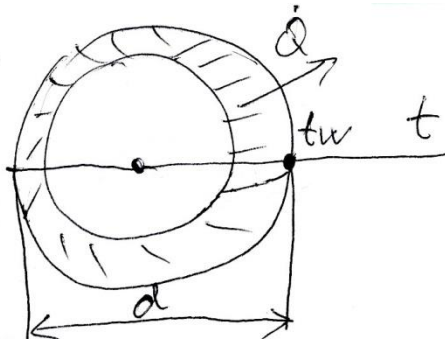


Presence of non-condensing inert in the vapor (air) makes  $\alpha$  smaller because it prevents vapor from flowing toward wall

**Table comparing typical values of  $\alpha$**

	$\alpha$
transfer to/from gas	$\sim 10^1 \text{ W m}^{-2} \text{ K}^{-1}$
transfer to/from liquid	$\sim 10^2 \text{ W m}^{-2} \text{ K}^{-1}$
bubble boiling	$\sim 10^3 \text{ W m}^{-2} \text{ K}^{-1}$
condensation	$\sim 10^4 \text{ W m}^{-2} \text{ K}^{-1}$

**Heat transfer in a cylinder (=pipe)**

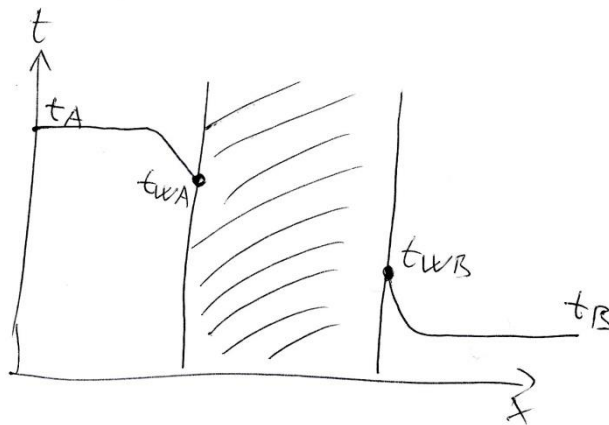


$$d\dot{Q} = \alpha(t_w - t)dA = \alpha(t_w - t)\pi ddL$$

$$= \pi \frac{(t_w - t)}{\frac{1}{\alpha d}} ; \frac{1}{\alpha d} \dots \text{resistance}$$

**Overall heat transfer**

This is a combination of heat transfer from fluid A to a wall, heat flow through the wall and heat transfer from the wall to fluid B.



By combining heat flow expressions and summing up all resistances we get:

$$d\dot{Q} = \frac{(t_A - t_B)}{\frac{1}{\alpha_A} + \sum_{i=1}^n \frac{\delta_i}{\lambda_i} + \frac{1}{\alpha_B}} dA = K(t_A - t_B)dA; K \dots \text{overall heat transfer coefficient (planar geom.)}$$

$$[K] = \text{W m}^{-2} \text{ K}^{-1}$$

For pipes (cylinders)

$$d\dot{Q} = \frac{\pi(t_A - t_B)}{\frac{1}{\alpha_A d_A} + \sum \frac{1}{2\lambda_i} \ln \frac{d_i}{d_{i-1}} + \frac{1}{\alpha_B d}} dL = K_L(t_A - t_B)dA; K_L \dots \text{overall heat transfer coef. (pipes)}$$

$$[K_L] = W m^{-1} K^{-1}$$

## Heat exchangers

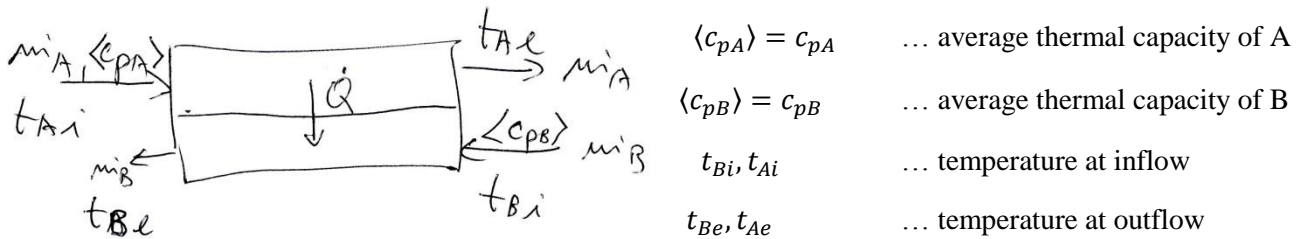
Used for:

- heating
- cooling
- boiling
- condensation

### Notation:

A ... hotter fluid

B ... cooler fluid

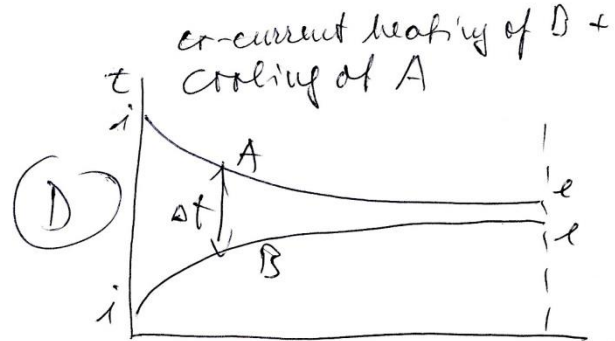
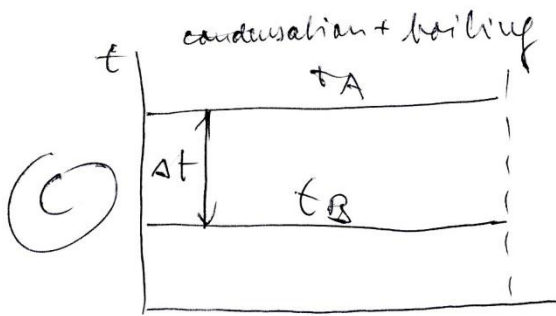
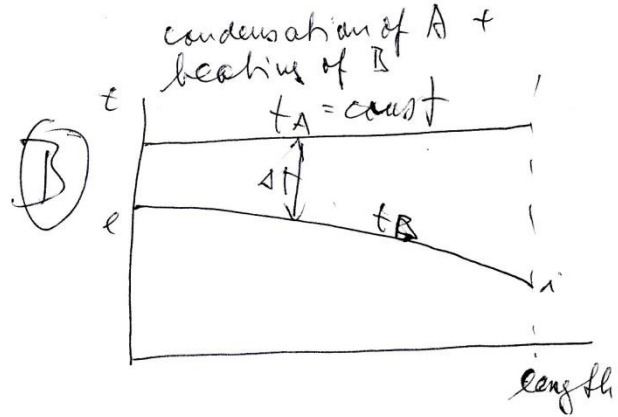
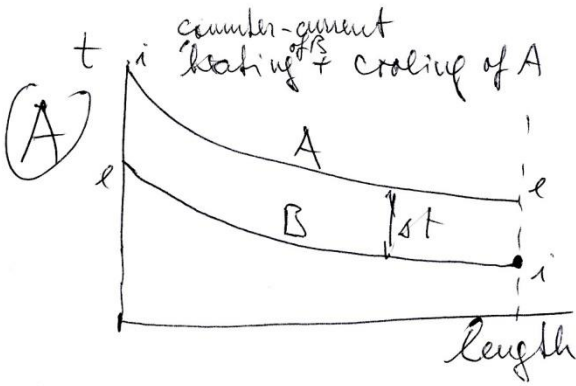


Most frequent arrangement is **counter-current** as shown in the figure, also possible is **co-current** or **cross-flow**.

### Geometry:

- **planar**: plate heat exchangers
- **cylindric**:
  - o tube-within-a-tube
  - o tubular exchangers with a bundle of tubes in a shell

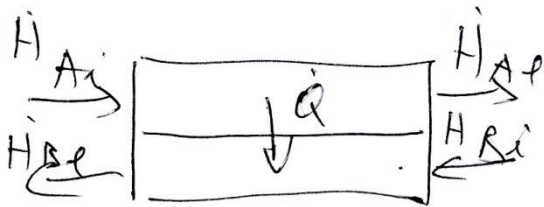
### Temperature profiles



- counter current heating of B + cooling of A
- condensation of A + heating of B
- condensation + boiling
- co-current heating of B + cooling of A

$\Delta t$  ... **driving force** – normally varies along the exchanger, constant for case C

### Enthalpy balance of counter-current heat exchanger



$$\dot{H}_{Ai} = \dot{H}_{Ae} + \dot{Q}$$

$$\dot{H}_{Bi} + \dot{Q} = \dot{H}_{Be}$$

$$\dot{H}_{Ai} + \dot{H}_{Bi} = \dot{H}_{Ae} + \dot{H}_{Be}$$

$$\dot{H}_{Ai} = \dot{m}_A c_{pA} (t_{Ai} - t_{ref}); \dot{H}_{Ae} = \dot{m}_A c_{pA} (t_{Ae} - t_{ref})$$

$$\dot{Q} = \dot{m}_A c_{pA} (t_{Ai} - t_{Ae}) = \dot{m}_B c_{pB} (t_{Be} - t_{Bi})$$

$$c_{pA} \text{ evaluated at } \langle t_A \rangle = \frac{t_{Ai} + t_{Ae}}{2}; c_{pB} \text{ evaluated at } \langle t_B \rangle = \frac{t_{Bi} + t_{Be}}{2}$$

Once  $\dot{Q}$  is calculated from the enthalpy balance, the rate equation for heat transfer can be used to calculate the area  $A$  or the length  $L$  of exchanger. However, because the driving force  $\Delta t$  varies along the exchanger, a differential enthalpy balance must be used in the first place + combined with the heat transfer equation:

$$d\dot{Q} = K \underbrace{(t_A - t_B)}_{\Delta t} dA = \dot{m}_A c_{pA} dt_A = \dot{m}_B c_{pB} dt_B$$

$K$  ... overall heat transfer coefficient for a plate exchanger

or

$$d\dot{Q} = K_L (t_A - t_B) dL = \dot{m}_A c_{pA} dt_A = \dot{m}_B c_{pB} dt_B$$

$K_L$  ... overall heat transfer coefficient for a tubular exchanger

**By integration we obtain three useful formulae:**

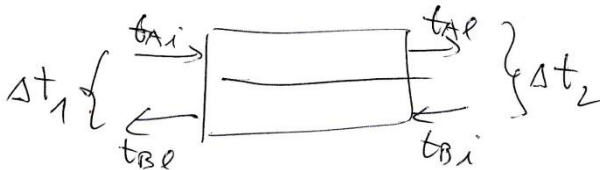
(i) the first one:

$$\dot{Q} = K \Delta t_{l.m.} A \quad \text{or} \quad \dot{Q} = K_L \Delta t_{l.m.} L$$

where the logarithmic mean temperature  $\Delta t_{l.m.}$  is

$$\Delta t_{l.m.} = \frac{\Delta t_1 - \Delta t_2}{\ln \frac{\Delta t_1}{\Delta t_2}} = \frac{(t_{Ai} - t_{Be}) - (t_{Ae} - t_{Bi})}{\ln \frac{t_{Ai} - t_{Be}}{t_{Ae} - t_{Bi}}}$$

The differences  $\Delta t_1$  and  $\Delta t_2$  are **driving forces** at both ends of the exchanger. They are called **approaches**.



(ii) another useful formula is

$$A = \frac{\dot{m}_A c_{pA}}{K} \frac{t_{Ai} - t_{Ae}}{\Delta t_{l.m.}} = \frac{\dot{m}_B c_{pB}}{K} \frac{t_{Be} - t_{Bi}}{\Delta t_{l.m.}} \quad \text{for plate heat exchangers}$$

$A_A, A_B$  ... are **areas of transfer units**

$N_A, N_B$  ... are **numbers of transfer units**

$$L = \frac{\dot{m}_A c_{pA}}{K_L} \frac{t_{Ai} - t_{Ae}}{\Delta t_{l.m.}} = \frac{\dot{m}_B c_{pB}}{K_L} \frac{t_{Be} - t_{Bi}}{\Delta t_{l.m.}} \quad \text{for tubular heat exchangers}$$

$L_A, L_B$  ... are **lengths of transfer units**

(iii) final formula is



$$\frac{1 - \zeta_{AB}\eta_A}{1 - \eta_A} = \exp(N_A(1 - \zeta_{AB}))$$

where

$$\zeta_{AB} = \frac{\dot{m}_A c_{pA}}{\dot{m}_B c_{pB}} \dots \text{thermal capacity ratio}$$

$$\eta_A = \frac{t_{Ai} - t_{Ae}}{t_{Ai} - t_{Bi}} = \frac{\text{temperature difference in phase A}}{\text{maximal temperature difference}} \dots \text{effectiveness}$$

This formula is useful, when the area  $A$  or length  $L$  are known but both output temperatures  $t_{Ae}$ ,  $t_{Be}$  are unknown. Then  $\eta_A$  can be evaluated from the formula iii) and from the  $\eta_A$  then  $t_{Ae}$ . The second unknown  $t_{Be}$  is then calculated from the enthalpy balance

$$\dot{m}_A c_{pA} (t_{Ai} - t_{Ae}) = \dot{m}_B c_{pB} (t_{Be} - t_{Bi})$$

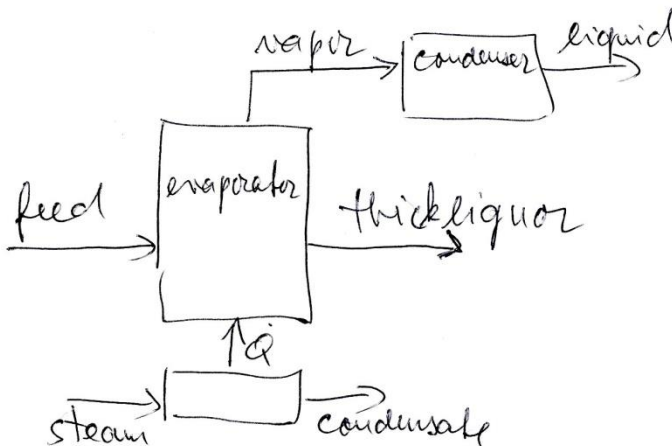
**Remark:**

If condensation or boiling occurs, formulae i) and ii) still apply, but the enthalpy balance is for example for condensation of A:

$$\dot{Q} = \dot{m}_A h_{cond} = \dot{m}_B c_{pB} (t_{Be} - t_{Bi})$$

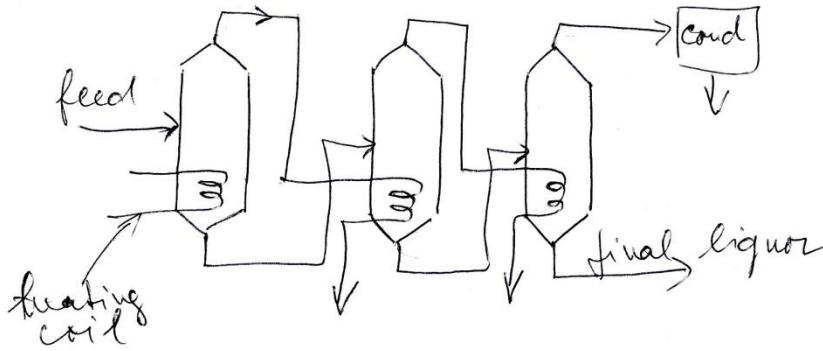
**Evaporation**

- removal of a liquid solvent from solution of a nonvolatile component by boiling and removal of the vapor
- typically boiling is caused by heating  $\rightarrow$  similar to a heat exchanger

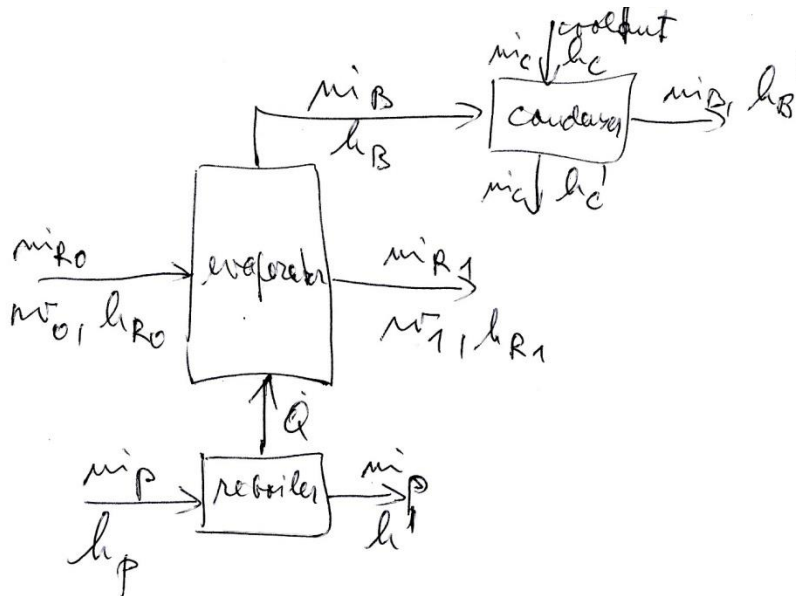


For more effective operation, vapor could be **compressed** and used for heating of the same unit or subsequent units.

For example: **co-current two-stage evaporator**



**Description of a single unit**



**Mass balance of the evaporator**

- total:  $\dot{m}_{R0} = \dot{m}_{R1} + \dot{m}_B$
- non-volatile component:  $w_0 \dot{m}_{R0} = w_1 \dot{m}_{R1}$

**Enthalpy balance of the evaporator**

$$\dot{Q} + \dot{m}_{R0} h_{R0} = \dot{m}_{R1} h_{R1} + \dot{m}_B h_B$$

**Enthalpy balance of the reboiler**

$$\dot{Q} = \dot{m}_p (h_p - h_i) = \dot{m}_p \Delta h_{cond}; \Delta h_{cond} \dots \text{condensation enthalpy per unit mass}$$

**Enthalpy balance of the condenser**

$$\dot{m}_p (h_p - h_B) = \dot{m}_c (h_c' - h_c) = \dot{Q}_{condenser}$$

The enthalpy  $h_{R1}$  is found at temperature  $t_1 = t_s^0 + \Delta t_e$

$t_s^0$  ... **boiling point** of the solvent

$\Delta t_e$  ... **temperature elevation** due to dissolved component

**Area** of the heat exchange in the reboiler is calculated from the heat transfer equation:

$$\dot{Q} = K(t_p - t_1)A$$

$t_p$  ... temperature of the steam in the reboiler

$t_1$  ... temperature in the evaporator

$K$  ... overall heat transfer coefficient

$\dot{Q}$  ... heat flow is calculated from the enthalpy balance